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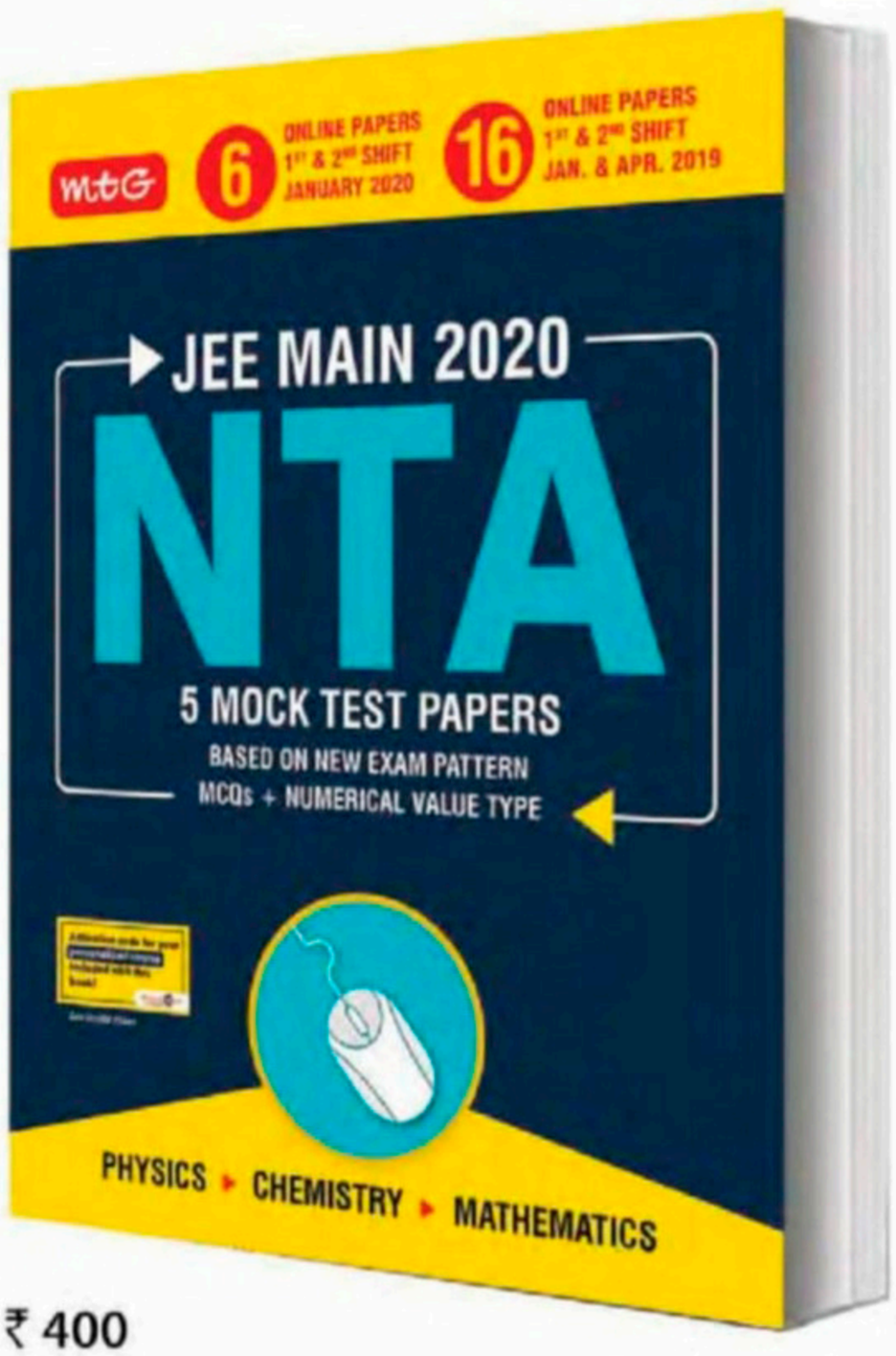

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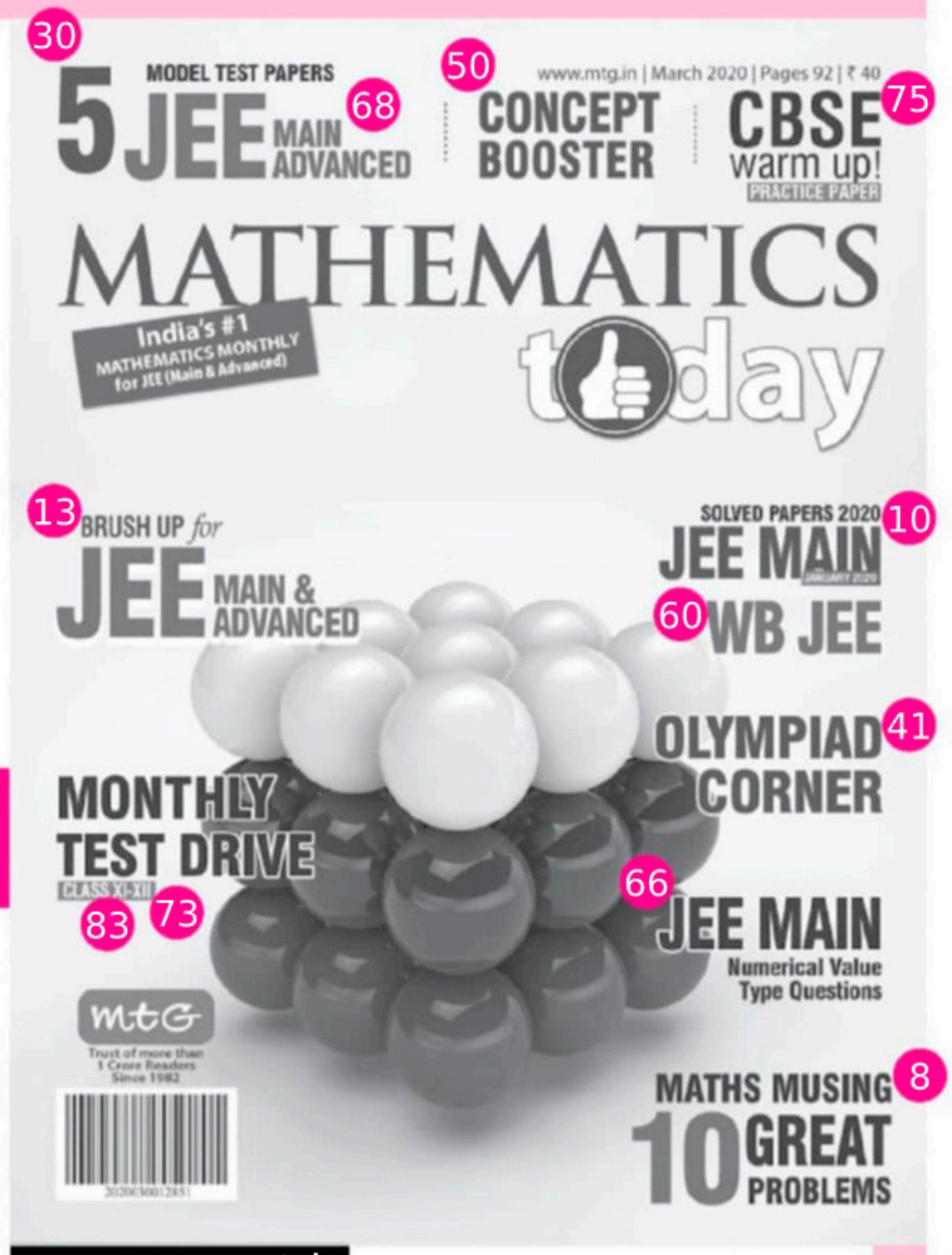

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new *pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our* readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 207

JEE MAIN

1. The value of $\sum_{i=0}^{n}\sum_{j=1}^{n} {}^nC_j\, {}^jC_i,\ i \le j$ is

(a) $3^n - 1$ (b) 0 (c) 2^n (d) $2^n - 1$

2. The length of the perpendicular drawn from origin to the plane passing through a point $\vec{c}$ and containing the line $\vec{r} = \vec{b} + \mu\vec{a}$ is

(a) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$ (b) $\dfrac{[\vec{c}\,\vec{b}\,\vec{a}]}{|\vec{b}\times\vec{a} + \vec{a}\times\vec{c}|}$

(c) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c}|}$ (d) $\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}|}$

3. Let $f(x) = ax^2 + bx + c$, $a, b, c \in I$. Let $f(1) = 0$, $f(7) \in (50, 60)$, $f(8) \in (70, 80)$, then $f(2) \in$

(a) $(-2, 0)$ (b) $(0, 10)$ (c) $(1, 12)$ (d) $(20, 30)$

4. The pair of lines $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ is rotated about the origin by $\pi/6$ in the anti-clockwise sense. The equation of the pair in the new position is

(a) $x^2 - \sqrt{3}\,xy = 0$ (b) $xy - \sqrt{3}y^2 = 0$

(c) $\sqrt{3}x^2 - xy = 0$ (d) $x^2 = y^2$

5. If $a\sin^2 x + b\cos^2 x = c$, $b\sin^2 y + a\cos^2 y = d$ and $a\tan x = b\tan y$, then $a^2/b^2 =$

(a) $\dfrac{(a-d)(c-a)}{(b-c)(d-b)}$ (b) $\dfrac{(a+d)(c+a)}{(b+c)(d+b)}$

(c) $\dfrac{(a-d)(b-a)}{(a-c)(c-b)}$ (d) $\dfrac{(d-a)(c-a)}{(b-c)(d-b)}$

JEE ADVANCED

6. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then which of the following is incorrect?

(a) $P(\overline{A \cup B}) = P(\bar{A})\,P(\bar{B})$

(b) $P(A|B) = P(A)$

(c) $P(\bar{A} \cup \bar{B}) = P(\bar{A}) + P(\bar{B})$

(d) $P(B|A) = P(B) - P(A)$

COMPREHENSION

Given $A(3 + 4i)$, $B(-4 + 3i)$ and $C(4 + 3i)$ be the vertices of ΔABC which is inscribed in a circle $S = 0$. Let AD, BE, CF are altitudes through A, B, C which meet the circle $S = 0$ at $D(z_1)$, $E(z_2)$ and $F(z_3)$ respectively.

7. Complex number z_1 is equal to

(a) $2\sqrt{6} + i$ (b) $1 + 2\sqrt{6}i$

(c) $3 - 4i$ (d) $-3 - 4i$

8. Orthocentre of triangle ABC is at

(a) $2 + 9i$ (b) $3 + 10i$

(c) $3 + 11i$ (d) none of these

NUMERICAL VALUE TYPE

9. The number of ordered pairs (m, n) $(m, n \in \{1, 2, \ldots, 20\})$ such that $3^m + 7^n$ is a multiple of 10, is

MATRIX MATCH

10. Match the following columns :

	Column-I		Column-II
P.	For a rectangular hyperbola $xy = c^2$ (c is purely imaginary). If a point on it is (a, α), where 'a' is a positive number, then α can take value	1.	3
Q.	If sides of a triangle are in A.P. and $(a - b + c)s = kb^2$ (where s is the semi perimeter), then k is equal to	2.	$\dfrac{1-\sqrt{3}}{2}$
R.	Radius of the circle having centre (3, 4) and touching the circle $x^2 + y^2 = 4$ can be	3.	$\dfrac{3}{2}$

	P	Q	R
(a)	1	2	3
(b)	1	3	2
(c)	3	1	2
(d)	2	3	1

See Solution Set of Maths Musing 206 on page no. 49

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Knowledge Quiz - 28

Let u, v, w be real numbers in geometric progression such that $u > v > w$. Suppose $u^{40} = v^n = w^{60}$. Find the value of n.

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JEE MAIN 2020

1. If $3x+4y=12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a \in R$, then the distance between the foci of the ellipse is

(a) $2\sqrt{7}$ (b) 4 (c) $2\sqrt{2}$ (d) $2\sqrt{5}$

2. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is $(102)m$, then m is equal to

(a) 5 (b) 10 (c) 20 (d) 25

3. The locus of the mid-points of the perpendiculars drawn from points on the line $x = 2y$ to the line $x = y$ is

(a) $3x - 2y = 0$ (b) $5x - 7y = 0$
(c) $2x - 3y = 0$ (d) $7x - 5y = 0$

4. Let $a_1, a_2, a_3, ...$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to

(a) −171 (b) −513 (c) $\frac{511}{3}$ (d) 171

5. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to

(a) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ (b) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$
(c) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (d) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

6. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is 1/4. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to

(a) $\frac{17}{8}$ (b) $\frac{17}{2}$ (c) $\frac{17}{4}$ (d) 4

7. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$, is

(a) $\log_e\sqrt{2}$ (b) $\log_e 2$
(c) $\log_e\left(\frac{3}{2}\right)$ (d) $\log_e\left(\frac{4}{3}\right)$

8. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta\in[0, 2\pi]$ is a real number, then an argument of $\sin\theta + i\cos\theta$ is

(a) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$
(c) $-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

9. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. Then $(AB)^2$ is equal to

(a) $\frac{32}{5}$ (b) $\frac{56}{5}$ (c) $\frac{52}{5}$ (d) $\frac{64}{5}$

10. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k, k \geq 1$, then which one of the following statements is not true?

(a) $p_3 = p_5 - p_4$
(b) $p_5 = 11$
(c) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
(d) $p_5 = p_2 \cdot p_3$

11. The number of ordered pairs (r, k) for which $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$, where k is an integer, is

(a) 2 (b) 3 (c) 4 (d) 6

12. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2+\frac{f(x)}{x^3}\right)=4$, then which one of the following is not true?

(a) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

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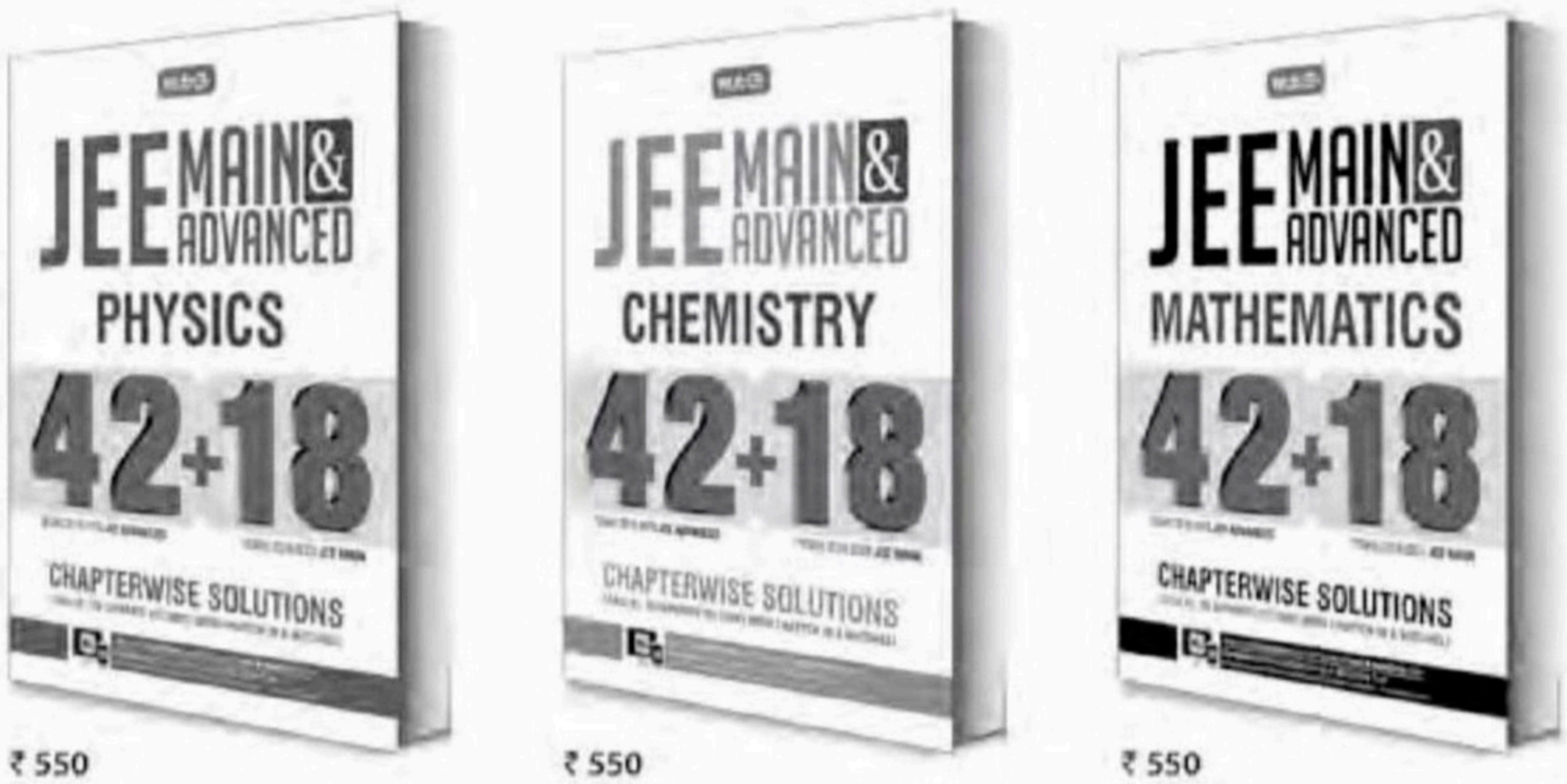

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(b) f is an odd function.
(c) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.
(d) $f(1) - 4f(-1) = 4$

13. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where $i, j = 1, 2, 3$. If the determinant of B is 81, then the determinant of A is
(a) 1/3 (b) 1/81 (c) 1/9 (d) 3

14. Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is
(a) $-e$ (b) $2 + e$ (c) $2 - e$ (d) 2

15. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0, 1]$ is
(a) $\frac{2}{3}$ (b) $\frac{4-\sqrt{7}}{3}$ (c) $\frac{4-\sqrt{5}}{3}$ (d) $\frac{\sqrt{7}-2}{3}$

16. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + ... + x^{10}$ is
(a) 420 (b) 330 (c) 120 (d) 210

17. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to
(a) $\frac{\pi}{9}$ (b) $\frac{\pi}{3} + \frac{1}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

18. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = 1/2$ is equal to
(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{\sqrt{5}}{2}$ (c) $-\frac{\sqrt{5}}{2}$ (d) $-\frac{\sqrt{5}}{4}$

19. The area (in sq. units) of the region $\{(x, y) \in R^2 \mid 4x^2 \le y \le 8x + 12\}$ is
(a) $\frac{125}{3}$ (b) $\frac{127}{3}$ (c) $\frac{128}{3}$ (d) $\frac{124}{3}$

20. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is
(a) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$
(b) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$
(c) If $A \subseteq C$, then $B \subset A$ or $D \subset B$
(d) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

21. If the foot of the perpendicular drawn from the point $(1, 0, 3)$ on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to _____.

22. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _____.

23. If the system of linear equations,
$x + y + z = 6$
$x + 2y + 3z = 10$
$3x + 2y + \lambda z = \mu$
has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

24. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by
$$f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$$
is continuous, then k is equal to _____.

25. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n \text{ is a multiple of } 2\}$ and $B = \{n \in X : n \text{ is a multiple of } 7\}$, then the number of elements in the smallest subset of X containing both A and B is _____.

ANSWER KEYS

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (c)
6. (a)	**7.** (b)	**8.** (d)	**9.** (d)	**10.** (d)
11. (c)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (b)
16. (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (b)
21. (4)	**22.** (54)	**23.** (13)	**24.** (5)	**25.** (29)

For complete solutions, refer to
MTG NTA JEE Main January 2020 Solved Papers.

COMIC CAPSULE

"My blood pressure is 180/90 which is mathematically equal to 2/1 which doesn't seem too high"

BRUSH UP

for JEE Main & Advanced 2020

Brush up your concepts to get high rank in JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year according to the requirements of these Entrance Tests.

Unit 9

Integration and its Applications

INDEFINITE INTEGRAL

- Integration is the reverse process of differentiation. If $f(x)$ and $g(x)$ are two functions such that $\frac{d}{dx}[f(x)] = g(x)$, then we say that $f(x)$ is integration of $g(x)$. We write it as $\int g(x)\,dx = f(x) + c$, where $g(x)$is called integrand, $f(x)$ is called the antiderivative of $g(x)$ or the primitive of $g(x)$ and c is known as constant of integration, which can assume any real value.

Properties of Indefinite Integrals

- $\int a\,\phi(x)\,dx = a\int \phi(x)dx,\ \forall\, a \in R$
- $\int (f(x) \pm g(x))\,dx = \int f(x)dx \pm \int g(x)\,dx$
  This can be generated to n number of functions.
- If $\int g(x)dx = f(x) + c$, then
  $$\int g(Ax+B)\,dx = \frac{f(Ax+B)}{A} + c,\ A \neq 0$$

METHODS OF INTEGRATION

(1) Integration by Substitution

Some Standard Substitutions

Expression	Substitution
$x^2 - a^2,\ \frac{1}{x^2-a^2},\ \frac{1}{\sqrt{x^2-a^2}}$ $\sqrt{x^2-a^2}$	$x = a\sec\theta$ or $x = a\operatorname{cosec}\theta$
$x^2 + a^2,\ \frac{1}{x^2+a^2},\ \frac{1}{\sqrt{x^2+a^2}}$ $\sqrt{x^2+a^2}$	$x = a\tan\theta$ or $x = a\cot\theta$
$a^2 - x^2,\ \frac{1}{a^2-x^2},\ \frac{1}{\sqrt{a^2-x^2}}$ $\sqrt{a^2-x^2}$	$x = a\sin\theta$ or $x = a\cos\theta$
$(\sqrt{x^2+a^2} \pm x)^n$ or $(x \pm \sqrt{x^2-a^2})^n$ or their reciprocals	$\sqrt{x^2+a^2} \pm x = t$ or $x \pm \sqrt{x^2-a^2} = t$
$\sqrt{(x-a)(b-x)},\ \sqrt{\frac{x-a}{b-x}},$ $\frac{1}{\sqrt{(x-a)(b-x)}}$	$x = a\cos^2\theta + b\sin^2\theta$ or $x = a\sin^2\theta + b\cos^2\theta$
$\sqrt{a+x},\ \sqrt{a-x},\ \frac{a \pm x}{a \mp x},$ $\sqrt{\frac{a \pm x}{a \mp x}}$ and their reciprocals	$x = a\cos\theta$ or $x = a\sin\theta$ Sometime $x = a\tan^2\theta$ or $x = a\cot^2\theta$ make the form more easier
$\sqrt{(x-a)(x-b)},\ \sqrt{\frac{x-a}{x-b}},$ $\frac{1}{\sqrt{(x-a)(x-b)}}$	$x = a\sec^2\theta - b\tan^2\theta$ or $x = a\operatorname{cosec}^2\theta - b\cot^2\theta$
$\left(\frac{x+b}{x+a}\right)^{\frac{1}{n}-1} \frac{1}{(x+a)^2}$ or $\left(\frac{x+b}{x+a}\right)^{1/n} \frac{1}{(x+a)(x+b)},$ $(n \in N, n > 1)$	$\frac{x+b}{x+a} = t$ or $\frac{1}{t}$, as the case exist.

By : **R. K. Tyagi**, Retd. Principal, HOD Maths, Samarth Shiksha Samiti, New Delhi

Integration of the form: $\int x^m(a+bx^n)^p\,dx, (p\neq 0)$, m, n, p are rationals. In this case the following cases arise

Case-1: If p is an integer:

- If p is a positive integer, then expand $(a + bx^n)^p$ by using binomial theorem and integrate.
- If p is a negative integer and $m, n \in Q$, then put $x = t^k$, where k is the L.C.M. of denominators of m, n.

Case-2: If $\frac{m+1}{n}$ is an integer and p is rational, then put $(a + bx^n) = t^k$, where k is the denominator of the fraction p.

Case-3: If $\frac{m+1}{n}+p$ is an integer, then put $a + bx^n = x^n \cdot t^k$, where k is the denominator of the fraction p.

(2) Integration by Parts

If $f(x)$ and $g(x)$ are two functions of x, then integral of their product is $\int f(x)\cdot g(x)\,dx = f(x)\int g(x)\,dx - \int\left(\frac{d}{dx}(f(x))\int g(x)\,dx\right)dx$

In order to choose the first function, we follow the letters of the word ILATE, (Inverse, Logarithmic, Algebraic, Trigonometric, Exponential).

(3) Integration by Partial Fractions

Let $f(x)$ and $g(x)$ be two polynomial, then $\frac{f(x)}{g(x)}(g(x)\neq 0)$ is called algebraic rational function.

If degree of $f(x)$ < degree of $g(x)$, then $\frac{f(x)}{g(x)}$ is called proper rational function.

Any proper rational function can be expressed as a sum of rational functions, where each rational number has a simple factor of $g(x)$, Rational functions are called partial fractions and this process of expressing $\frac{f(x)}{g(x)}$ into sum of rational functions is called partial functions. Following are the rules to integrate using partial fractions :

- If $g(x) = (x - a_1)(x - a_2) \ldots (x - a_n)$, then $\frac{f(x)}{g(x)}=\frac{A_1}{x-a_1}+\frac{A_2}{x-a_2}+\ldots+\frac{A_n}{x-a_n}$, where $A_1, A_2, \ldots, A_n$ are constants which are obtained by equating numerator of L.H.S. and R.H.S. by putting the values $x = a_1, a_2, \ldots, a_n$.
- If $g(x)=(x-a)^k\cdot(x-a_1)(x-a_2)\ldots(x-a_n)$, then $\frac{f(x)}{g(x)}=\frac{A_1}{(x-a)}+\frac{A_2}{(x-a)^2}+\ldots+\frac{A_k}{(x-a)^k}+\frac{B_1}{x-a_1}+\frac{B_2}{x-a_2}+\ldots+\frac{B_n}{x-a_n}$, where $A_1, A_2, \ldots, A_k, B_1, B_2, \ldots, B_n$ are constants obtained by equating numerator of both sides and by equating like terms of both sides.
- If $g(x)$ has quadratic factor (irreducible), say $ax^2 + bx + c$, then partial fraction of $ax^2 + bx + c$ is taken as $\frac{Bx+C}{ax^2+bx+c}$.
- If $g(x)$ has irreducible factor, which are repeating, then partial fraction for $(ax^2 + bx + c)^k$ is taken as $\frac{Bx+c}{ax^2+bx+c}+\frac{Dx+E}{(ax^2+bx+c)^2}+\ldots.+\frac{Px+Q}{(ax^2+bx+c)^k}$
- If degree of $f(x)$ is greater or equal to the degree of $g(x)$, we first write $\frac{f(x)}{g(x)}=h(x)+\frac{p(x)}{g(x)}$, where $h(x)$ is a polynomial and $p(x)$ is also a polynomial of degree less than the degree of $g(x)$.

TRIGONOMETRIC INTEGRATION

- $\int\frac{a\sin x+b\cos x}{c\sin x+d\cos x}\,dx$

  In this integral, we express numerator as : A(denominator) + B(derivative of denominator) and find the values of A and B by equating the coefficients of sin$x$, cos$x$ on both sides.
- $\int\frac{a\sin x+b\cos x+R_1}{c\sin x+d\cos x+R_2}\,dx$

  In this integral we express numerator as : A (denominator) + B(derivative of denominator) + k and find the values of A, B and k by equating the coefficients of sin$x$, cos$x$ and the constant term.
- Integrals of the form $\int\frac{dx}{a\sin^2 x+b\cos^2 x}$, $\int\frac{dx}{a+b\sin^2 x}$, $\int\frac{dx}{a+b\cos^2 x}$, $\int\frac{dx}{(a\sin x+b\cos x)^2}$, $\int\frac{dx}{a\sin^2 x+b\cos^2 x+c}$

  In such type of integrals, first multiply and divide the denominator by $\cos^2 x$ and put $\tan x = t$, to get the standard form.
- Integrals of the form: $\int\sin^m x\cos^n x\,dx\ \forall\ m, n\in N$

(i) If any one of m and n is odd, then make the substitution for even power.
(ii) If m, n both are odd, then put either $\cos x = t$ or $\sin x = t$.
(iii) If m and n both are even, then use trigonometrical identity.

$$\sin^2 x = \frac{1-\cos 2x}{2}, \cos^2 x = \frac{1+\cos 2x}{2}.$$

(iv) If m and n are rational numbers and $\frac{m+n-2}{2}$ is a –ve integer, then put $\tan x = t$ or $\cot x = t$.

DEFINITE INTEGRAL

Fundamental Theorem of Integral Calculus

- Let $g(x)$ be the primitive or antiderivative of a function $f(x)$, which is differentiable on $[\alpha, \beta]$ *i.e.*, $\frac{d}{dx}(g(x)) = f(x)$. then the definite integral of $f(x)$ over closed interval $[\alpha, \beta]$ is denoted by and defined by $\int_\alpha^\beta f(x)\,dx = [g(x)]_\alpha^\beta = g(\beta) - g(\alpha)$, where α, β are known as the limits of integration. α is called lower limit and β is called upper limit. The interval $[\alpha, \beta]$ is called interval of integration.

Definite Integral as the Limit of a Function

- Let $f(x)$ be a continuous function on $[\alpha, \beta]$, then

$$\int_\alpha^\beta f(x)\,dx = \lim_{h\to 0} h[f(\alpha) + f(\alpha+h) + \ldots + f(\alpha+(n-1)h)], \text{ where } h = \frac{\beta-\alpha}{n}$$

$$= \lim_{n\to\infty} h\sum f(\alpha+rh), \text{ where } h = \frac{\beta-\alpha}{n}, 0 \le r \le n-1$$

- If $\alpha = 0$, $\beta = 1$, $nh = 1$, then we have

$$\int_0^1 f(x)\,dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(rh) = \lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$$

- $\int_\alpha^\beta f(x)\,dx = (\beta-\alpha)\int_0^1 f(\alpha+(\beta-\alpha)x)\,dx$

Leibnitz Rule for Differentiation under the Integral Sign

- Let $\phi(x)$ and $\psi(x)$ be two functions defined on $[\alpha, \beta]$ and differentiable at a point $x \in]\alpha, \beta[$ and $f(x, t)$ is continuous function, then

$$\frac{d}{dx}\left[\int_{\phi(x)}^{\psi(x)} f(x,t)\,dt\right] = \int_{\phi(x)}^{\psi(x)} \frac{\partial}{\partial x} f(x,t)\,dt + \left[\frac{d}{dx}(\psi(x))\right] f(x,\psi(x)) - \left[\frac{d}{dx}(\phi(x))\right] f(x,\phi(x))$$

where $f(x, t)$ is an implicit function.

- For explicit function, if $\phi(x)$ and $\psi(x)$ are defined on $[\alpha, \beta]$ and differentiable at a point $x \in]\alpha, \beta[$ and $f(t)$ is continuous on $[\phi(\alpha), \phi(\beta)]$, then

$$\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)\,dt\right) = \left\{\frac{d}{dx}(\psi(x))\right\} f(\psi(x)) - \left\{\frac{d}{dx}(\phi(x))\right\} f(\phi(x))$$

Properties of Definite Integrals

- $\int_a^b f(x)dx = \int_a^b f(t)\,dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,\ a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)\,dx = \begin{cases} 2\int_0^{a/2} f(x)\,dx, \text{ if } f(a-x) = f(x) \\ 0, \text{ if } f(a-x) = -f(x) \end{cases}$
- $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x) = f(x), \\ & i.e., f(x) \text{ is even} \\ 0, \text{ if } f(-x) = -f(x) & \\ & i.e., f(x) \text{ is odd} \end{cases}$
- If $f(t)$ is an odd function, then $g(x) = \int_a^x f(t)\,dt$ is an even function.

Definite Integral of Periodic Function

If $f(x)$ is a periodic function with period T, then

- $\int_\alpha^{\alpha+nT} f(x)\,dx = n\int_0^T f(x)\,dx, n \in I$
- $\int_{\alpha T}^{\beta T} f(x)\,dx = (\beta-\alpha)\int_0^T f(x)\,dx,\ \alpha, \beta \in I$
- If $f(x)$ is a periodic function with period T, then $\int_\alpha^{\alpha+T} f(x)\,dx$ is independent of α

i.e., if $\phi(\alpha) = \int_\alpha^{\alpha+T} f(x)\,dx$, then $\frac{d}{d\alpha}(\phi(\alpha)) = 0$

i.e., $\phi(\alpha)$ is a constant function.

- If $f(x)$ is an odd periodic function defined on the interval $\left[-\frac{T}{2}, \frac{T}{2}\right]$, where T is period of $f(x)$, then

$g(x)=\int_{\alpha}^{x} f(t)\,dt$ is periodic with period T

i.e., $g(x+T)=g(x)$.

- If $f(x)$ is an odd periodic function with period T, then $f(x)=\int_{\alpha}^{x} f(t)\,dt$ is an even function with period T.

Standard Results on Definite Integral

- $\int_0^{\pi/2} \sin^n x\,dx=\int_0^{\pi/2} \cos^n x\,dx$

$$=\begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}, & \text{if } n \text{ is even, } n\in N\\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}\cdots\frac{2}{3}, & \text{if } n \text{ is odd, } n\in N\end{cases}$$

- $\int_0^{\pi/2} \sin^m x\cos^n x\,dx=$

(i) $\frac{[(m-1)(m-3)\ldots5\cdot3\cdot1][(n-1)(n-3)\ldots5\cdot3\cdot1]}{(m+n)(m+n-2)\ldots6\cdot4\cdot2}\times\frac{\pi}{2}$

(when m and n both are even positive integers)

(ii) $\frac{[(m-1)(m-3)\ldots5\cdot3\cdot1][(n-1)(n-3)\ldots6\cdot4\cdot2]}{(m+n)(m+n-2)\ldots5\cdot3\cdot1}$

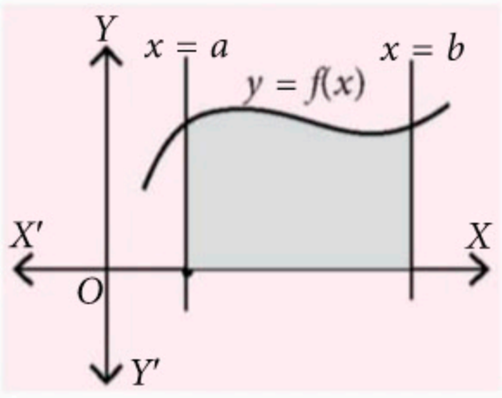
(when m is even and n is odd positive integer)

(iii) $\frac{[(m-1)(m-3)\ldots6\cdot4\cdot2][(n-1)(n-3)\ldots5\cdot3\cdot1]}{(m+n)(m+n-2)\ldots5\cdot3\cdot1}$

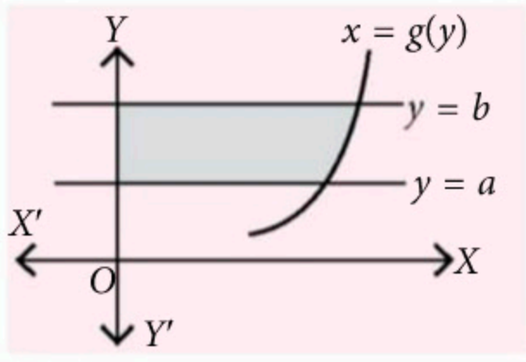
(when m is odd and n is even positive integer)

AREA UNDER THE CURVES

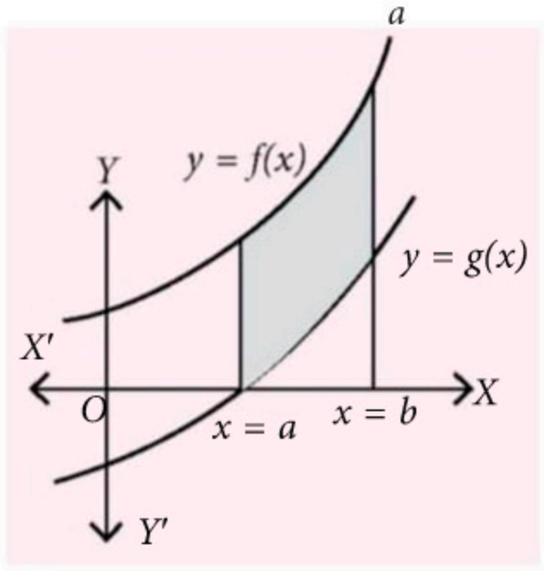
- The area bounded by curve $y=f(x)$ and x-axis between $x=a$ and $x=b$ is given by $\int_a^b |f(x)|\,dx$



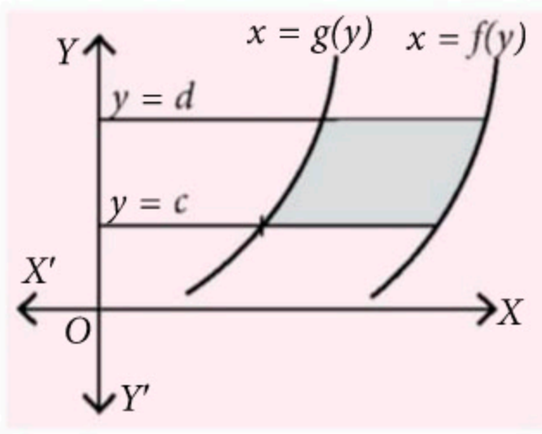
- The area bounded by the curve $x=g(y)$ and y-axis and between $y=a$ and $y=b$ is given by $\int_a^b |g(y)|\,dy$



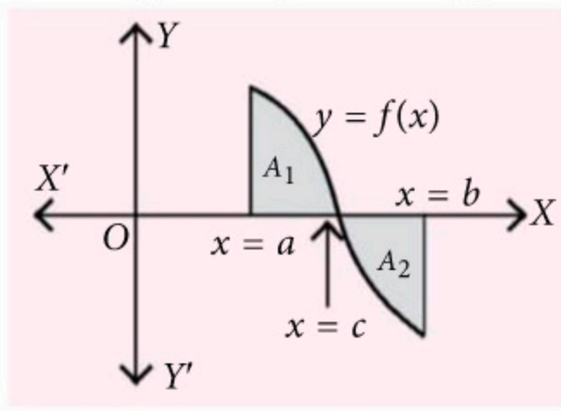
- The area bounded by the curve $y=f(x)$, $y=g(x)$ between $x=a$, $x=b$ is $\int_a^b |f(x)-g(x)|\,dx$



- The area bounded by the curves $x=f(y)$ and $x=g(y)$ between $y=c$ and $y=d$ is $\int_c^d |f(y)-g(y)|\,dy$



- If some region bounded by the curve lying above x-axis and some region lying below x-axis, then its count of whole area will be the sum of their absolute area of shaded portion, which is given by



$$A=\left|\int_a^c f(x)\,dx\right|+\left|\int_c^b f(x)\,dx\right|$$

PROBLEMS

Single Option Correct Type

1. Let $f(z)=\int_2^z \frac{dx}{6+x^4}$, then

(a) $f(3)<\frac{1}{22}$ (b) $f(3)>\frac{1}{22}$

(c) $f(3)>1$ (d) $f(3)=\frac{1}{22}$

2. If $f(t)=\int_0^t \frac{dx}{\sqrt{1+x^3}}$ and $h(t)$ be the inverse of f, then

(a) $h''(t) = 2$ (b) $h^2(t) = \frac{2}{3}h''(t)$

(c) $h''(t) = \frac{h'(t)}{2}$ (d) none of these

3. If $\int \frac{x^2-1}{x^4\sqrt{2x^6-3x^2+2}}dx = \frac{\sqrt{2x^6-3x^2+2}}{k\cdot x^{k/2}}+c$,

then k equals

(a) 3 (b) 4 (c) 6 (d) 12

4. If $\int \frac{dx}{x^4(1+x^4)^{1/4}} = \alpha(1 + x^{-4})\beta + c$, then the quadratic equation whose roots are α, β (where $\alpha, \beta \in R$), is

(a) $12x^2 + 5x - 3 = 0$ (b) $12x^2 - 5x - 3 = 0$

(c) $12x^2 - 5x + 3 = 0$ (d) $12x^2 + 5x + 3 = 0$

5. The integral $\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17}dx$ equals

(a) $\int_0^{\log(1+\sqrt{2})} (e^t - e^{-t})^{17}dt$

(b) $\int_0^{\log(1+\sqrt{2})} 2(e^t - e^{-t})^{17}dt$

(c) $\int_0^{\log(1+\sqrt{2})} 2(e^t + e^{-t})^{16}dt$

(d) $\int_0^{\log(1+\sqrt{2})} (e^t - e^{t})^{16}dt$

6. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x), \forall x \in [-1, 2]$. If $R_1 = \int_{-1}^{2} x\, f(x)dx$ and R_2 is the area of region bounded by $y = f(x)$, $x = -1$ and $x = 2$ and the x-axis, then

(a) $R_1 = 2R_2$ (b) $R_2 = 2R_1$

(c) $R_1 = 3R_2$ (d) $R_2 = 3R_1$

7. If $\int_0^{\pi/4} \sec^2\theta \sin\theta d\theta = \int_0^{\alpha} \frac{dx}{\sqrt{x}+\sqrt{x+\alpha}}$, then α equals

(a) 25/16 (b) 16/25 (c) 9/25 (d) 9/16

8. If $\int \frac{\cos 13x - \cos 14x}{1+2\cos 9x}dx = \frac{\sin k_1 x}{k_1} - \frac{\sin k_2 x}{k_2} + c$,

then the value of $k_1 + k_2$ is

(a) 9 (b) 13 (c) 14 (d) 18

9. $\int (x^m + 2x^{2m} + 3x^{3m})(1 + x^m + x^{2m})^{\frac{1}{m}}dx$ equals

(a) $\frac{(x^m + 2x^{2m} + 3x^{3m})^{m+1}}{m+1} + c$

(b) $\frac{(x^m + x^{2m} + x^{3m})^{\frac{m+1}{m}}}{m+1} + c$

(c) $\frac{(1 + x^m + x^{2m})^{\frac{m+1}{m}}}{m+1}$ (d) none of these

10. Let f and g be two symmetric function about the line $x = a$ and $h(x)$ be a function such that $2h(x) - 3h(2a - x) = 4$, then value of $\int_{-1}^{1} \left(\frac{\int_0^{2a} f(x)g(x)h(x)dx}{\int_0^{2a} f(x)\cdot g(x)dx} \right) dx$ is

(a) -4 (b) -6 (c) 8 (d) -8

11. If $\int \frac{\operatorname{cosec}^2 x - 2020}{(\cos x)^{2020}}dx = \frac{h(x)}{(\cos x)^{2020}} + c$, then value of $h\left(\frac{\pi}{6}\right)$ is

(a) 0 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $(1/\sqrt{3})$

12. If $f(x) = \sin(2\sin^{-1}(\cos x))$, then $\int_0^{\pi} xf(x)\sin 2x\, dx$ is

(a) $\frac{\pi^3}{4}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^3}{8}$

13. Let $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + t\cos x)f(t)\, dt$, then the value of $\int_0^{\pi/2} f(x)dx$ equals

(a) 2 (b) -2 (c) -1 (d) 1

More Than One Option(s) Correct Type

14. Let $A(x) = \int \frac{dx}{e^{3x} + 15e^{-3x} + 9e^{-9x}}$,

$B(x) = \int \frac{dx}{e^{9x} + 15e^{3x} + 9e^{-3x}}$ and

$M(x) = A(x) - 3B(x) = \frac{1}{\lambda}\tan^{-1}(f(x)) + c, c \in R$, then

(a) $\lambda = 9$ (b) $f(x) = \frac{e^{3x} + 3e^{-3x}}{3}$

(c) $27f(\log 3) = 244$ (d) $24f(\log 2) = 67$

15. If $I_{m-2,\, n+2} = \int \sin^{m-2}x \cos^{n+2}x\, dx$ and

$I_{m,n} = \frac{-\sin^{m-1}x\cos^{n+1}x}{n+1} + f(m,n)\, I_{m-2,\, n+2}$, then

(a) $f(5,\ 4)=4/5$ (b) $f(4,\ 5)=1/2$
(c) $f(2,2)+f(2,3)=7/12$ (d) $f(2,\ 2)+f(3,\ 2)=1$

16. Let f and g are integrable function $\forall$ real values and its subset, satisfy (i) f is odd, g is even (ii) $g(x) = f(x + 2020)$, then

(a) $f(x - 2020) = g(x)$ (b) $f(x - 2020) = -g(x)$

(c) $\int_0^{2020} f(t)dt = \int_0^{2020} g(2020-t)dt$

(d) $\int_0^{2020} f(t)dt = -\int_0^{2020} g(2020-t)dt$

17. Let $f(x)=\dfrac{\sin^m x}{1+a^x}$, $a>0$ and m is the even positive integer and $\int_{-\pi}^{\pi} f(x)\,dx=\alpha$, then

(a) α is independent of a

(b) for $m=4, \alpha=\dfrac{3\pi}{8}$ (c) for $m=2, \alpha=\dfrac{\pi}{2}$

(d) $f(x) + f(-x)$ is an even function

18. The area bounded by the ellipse $\dfrac{x^2}{36}+\dfrac{y^2}{25}=1$ (in. sq. unit) is equal to

(a) 36π (b) 30π
(c) 25π
(d) area bounded by $\dfrac{x^2}{25}+\dfrac{y^2}{36}=1$

19. Let A_1 and A_2 be areas lying above and below x-axis respectively and bounded by the curve $y = 2x - x^2$ and line $y = -x$, then

(a) $A_1+A_2=\dfrac{9}{2}$ sq. units (b) $A_1 > A_2$

(c) $A_2 > A_1$ (d) $\left|\dfrac{A_1}{A_2}\right|=\dfrac{8}{19}$

20. If $\int \dfrac{x^x(x^{4x}+1)(\log x+1)}{1+x^{6x}}dx=\sum_{r=1}^{3} a_r \tan^{-1} x^{rx}+c$, then

(a) $a_1 \cdot a_2 \cdot a_3 = 0$ (b) $a_2 < a_3 < a_1$

(c) $\sum_{r=1}^{3} ra_r=2$ (d) $a_1\ a_3 > 0$

Comprehension Type

Paragraph for Q.No. 21 to 24

Let $S_1 : y = x - 1\ \forall\ x \geq 1$
$S_2 : y = -(x - 1)\ \forall\ x \leq 1$
$S_3 : y = 3 - |x|\ \forall\ x \geq 0$
$S_4 : y = 3 - |x|\ \forall\ x \leq 0$

21. Area bounded by the curves S_1 and S_3 with x-axis (in sq. units) is

(a) 1 (b) 2 (c) 3 (d) 4

22. Area bounded by the curves S_2 and S_4 with x-axis (in sq. units) is

(a) 3 (b) 4 (c) 5 (d) 6

23. Area bounded by the curves S_3 and S_4 with x-axis (in sq. units)

(a) 11/2 (b) 9/2 (c) 7/2 (d) 9

24. Area bounded by all the curves S_1, S_2, S_3 and S_4 (in sq. units) is

(a) 9 (b) 5 (c) 4 (d) 1

Matrix Match Type

25. Match the following :

	Column-I		Column-II
P.	If $m = n$ where m, n are positive integers, then $\int_0^{\pi} \sin mx\ \sin nx\ dx =$	1.	0
Q.	If $\int_{\pi/6}^{\pi/3} \dfrac{\sin^{2020} x}{\cos^{2020} x+\sin^{2020} x}dx=\dfrac{k}{4}$, then $k =$	2.	$\pi/2$
R.	If $\int_0^{\infty} \dfrac{\sin x}{x}dx=\dfrac{\pi}{2}$, then $\int_0^{\infty} \dfrac{\sin^3 x}{x}dx =$	3.	$\pi/6$
S.	If $\int_0^{\pi/2} \sin 2x \log(\tan x)\,dx = k$, then $k =$	4.	$\pi/4$
		5.	$\pi/3$

Numerical Value Type

26. If $\int \dfrac{4x^3+3x^2}{x^8+2x^7+x^6+4x^4+4x^3+13}dx$ $=\dfrac{\alpha}{\beta}\tan^{-1}(Ax^4+Bx^3+C)+D$, then value of $(\alpha+\beta)^4\cdot\left(\dfrac{A+B}{C}\right)^4$ is equal to

27. Let $f(x)$ be a differentiable function satisfying $f(x) = x^2+\int_0^x e^{-t} f(x-t)dt$, then value of $f(3)$ is equal to

28. The value of $\dfrac{\int_0^1 (1-x^5)^{404}\,dx}{\int_0^1 (1-x^5)^{403}\,dx} = \lambda$, then 2021λ equals

29. If $A_1 = \int_0^{3\pi} f(\cos^2 x)dx$ and $A_2 = \int_0^{\pi} f(\cos^2 x)dx$, then the value of $\dfrac{A_1}{A_2}$ equals

30. The value of $\int_{-2}^{2} \dfrac{3x^2}{1+e^x}\,dx$ is equal to

SOLUTIONS

1. (a) : $f(z)=\int_2^z \dfrac{dx}{6+x^4} \quad \therefore \quad f'(z)=\dfrac{1}{6+z^4}$

By L.M.V. theorem, $f'(c)=\dfrac{f(3)-f(2)}{3-2}$ for some $c \in (2, 3)$

$\Rightarrow f'(c) = f(3)\ [\because f(2) = 0] \Rightarrow \dfrac{1}{6+c^4} = f(3)$

Now $2 < c < 3 \Rightarrow 2^4 < c^4 < 3^4$

$\Rightarrow 22 < 6 + c^4 < 87$

$\Rightarrow \dfrac{1}{87} < \dfrac{1}{6+c^4} < \dfrac{1}{22} \Rightarrow \dfrac{1}{87} < f(3) < \dfrac{1}{22}$

2. (b) : Given, $f(t)=\int_0^t \dfrac{dx}{\sqrt{1+x^3}}$ and $f(h(t)) = t$

$\therefore \ f(h(t))=\int_0^{h(t)} (1+x^3)^{-\frac{1}{2}}\,dx \Rightarrow t=\int_0^{h(t)} (1+x^3)^{-\frac{1}{2}}\,dx$

Using Leibnitz's theorem, we have

$1 = h'(t)(1 + h^3(t))^{-(1/2)} \Rightarrow h'(t) = (1 + h^3(t))^{1/2}$

$\Rightarrow (h'(t))^2 = 1 + h^3(t)$

$\Rightarrow 2h'(t) \cdot h''(t) = 3h^2(t)h'(t) \Rightarrow h^2(t)=\dfrac{2}{3}h''(t)$

3. (c) : Let $I=\int \dfrac{x^2-1}{x^4\sqrt{2x^6-3x^2+2}}\,dx$

$=\int \dfrac{x^2-1}{x^4\cdot x^3\sqrt{2-\frac{3}{x^4}+\frac{2}{x^6}}}\,dx = \int \dfrac{\left(\frac{1}{x^5}-\frac{1}{x^7}\right)}{\sqrt{2-\frac{3}{x^4}+\frac{2}{x^6}}}\,dx$

Put $2-\dfrac{3}{x^4}+\dfrac{2}{x^6}=t \Rightarrow 12\left(\dfrac{1}{x^5}-\dfrac{1}{x^7}\right)dx = dt$

$\therefore I=\dfrac{1}{12}\int \dfrac{dt}{\sqrt{t}} = \dfrac{\sqrt{2-\frac{3}{x^4}+\frac{2}{x^6}}}{6}+c = \dfrac{\sqrt{2x^6-3x^2+2}}{6x^3}+c$

$= \dfrac{\sqrt{2x^6-3x^2+2}}{kx^{k/2}}+c$ (Given) $\Rightarrow k = 6$

4. (b) : Let $I=\int \dfrac{dx}{x^4(1+x^4)^{1/4}} = \int \dfrac{dx}{x^5(1+x^{-4})^{1/4}}$

Putting $1 + x^{-4} = t^4$ in above equation, we get

$I=-\int t^2dt = -\dfrac{t^3}{3}+c = -\dfrac{1}{3}(1+x^{-4})^{\frac{3}{4}}+c$

$= \alpha(1+x^{-4})^{\beta}+c$ [Given] $\therefore \ \alpha=-\dfrac{1}{3}, \beta=\dfrac{3}{4}$

$\therefore$ The required quadratic equation is

$\left(x+\dfrac{1}{3}\right)\left(x-\dfrac{3}{4}\right)=0 \Rightarrow 12x^2 - 5x - 3 = 0$

5. (c) : Let $\operatorname{cosec}x + \cot x = e^t$ ...(i)

and $\operatorname{cosec}x - \cot x = e^{-t}$ ...(ii)

$\therefore \ \log(\operatorname{cosec}x + \cot x) = t$

On adding (i) and (ii), we get

$e^t + e^{-t} = 2\operatorname{cosec}x$ ...(iii)

On differentiating (i), we get

$-\operatorname{cosec}x\,(\operatorname{cosec}x + \cot x)dx = e^t\,dt$

$\Rightarrow 2\operatorname{cosec}x\,dx = -2dt$

For limits, as we assume $\log(\operatorname{cosec}x + \cot x) = t$

At $x=\dfrac{\pi}{4}$, $t=\log(1+\sqrt{2})$; At $x=\dfrac{\pi}{2}$, $t=\log(1+0)=0$

$\therefore \ \int_{\pi/4}^{\pi/2} (2\operatorname{cosec}x)^{17}\,dx = \int_{\pi/4}^{\pi/2} (2\operatorname{cosec}x)^{16}(2\operatorname{cosec}x)dx$

$= -\int_{\log(1+\sqrt{2})}^{0} 2(e^t+e^{-t})^{16}dt = 2\int_0^{\log(1+\sqrt{2})} (e^t+e^{-t})^{16}dt$

6. (b) : $\because f(x) = f(1 - x), \forall\ x \in [-1, 2]$

$\Rightarrow R_1=\int_{-1}^{2} xf(x)dx=\int_{-1}^{2}(1-x)f(1-x)\,dx$

$=\int_{-1}^{2} f(1-x)dx-\int_{-1}^{2} xf(1-x)dx=\int_{-1}^{2} f(x)dx-\int_{-1}^{2} xf(x)dx$

$\Rightarrow R_1 = R_2 - R_1 \Rightarrow R_2 = 2R_1$

7. (d) : Given $\int_0^{\pi/4} \sec^2\theta\cdot\sin\theta\,d\theta=\int_0^{\alpha}\dfrac{dx}{\sqrt{x}+\sqrt{x+\alpha}}$

$$\Rightarrow \int_{1/\sqrt{2}}^{1} z^{-2}dz = \frac{1}{\alpha}\int_{0}^{\alpha}(\sqrt{x+\alpha}-\sqrt{x})dx$$

(By putting $\cos\theta = z$)

$$\Rightarrow (\sqrt{2}-1) = \frac{4}{3}\sqrt{\alpha}(\sqrt{2}-1) \Rightarrow \sqrt{\alpha} = \frac{3}{4} \Rightarrow \alpha = \frac{9}{16}$$

8. (a) : We have, $\int \frac{\cos 13x - \cos 14x}{1+2\cos 9x}dx$

$$= \int \frac{2\sin\frac{27x}{2}\cdot\sin\frac{x}{2}}{\left[1+2\left(1-2\sin^2\frac{9x}{2}\right)\right]}\cdot\frac{\sin\frac{9x}{2}}{\sin\frac{9x}{2}}dx$$

$$= \int \frac{2\sin\frac{27x}{2}\cdot\sin\frac{x}{2}\cdot\sin\frac{9x}{2}}{3\sin\frac{9x}{2}-4\sin^3\frac{9x}{2}}dx = \int 2\sin\frac{x}{2}\cdot\sin\frac{9x}{2}dx$$

$$= \int(\cos 4x - \cos 5x)dx = \frac{\sin 4x}{4} - \frac{\sin 5x}{5} + c$$

$\therefore\ k_1 + k_2 = 9$

9. (b) : $\int (x^m + 2x^{2m} + 3x^{3m})(1+x^m+x^{2m})^{\frac{1}{m}}dx$

$$= \int x(x^{m-1} + 2x^{2m-1} + 3x^{3m-1})(1+x^m+x^{2m})^{\frac{1}{m}}dx$$

$$= \int (x^{m-1} + 2x^{2m-1} + 3x^{3m-1})(x^m + x^{2m} + x^{3m})^{\frac{1}{m}}dx$$

$$= \frac{1}{m}\int f'(t)(f(t))^{\frac{1}{m}}dt \left[\text{Putting } f(t) = x^m + x^{2m} + x^{3m}\right]$$

$$= \frac{1}{(m+1)}(x^m + x^{2m} + x^{3m})^{\frac{m+1}{m}} + c$$

10. (d) : $\because$ f and g are symmetric about the line $x = a$

$\therefore\quad f(a - x) = f(a + x)$ and $g(a - x) = g(a + x)$

Replacing x by $x + a$, we get

$f(-x) = f(x + 2a)$ and $g(-x) = g(x + 2a)$

Replacing x by $-x$, we get

$f(x) = f(2a - x)$ and $g(x) = g(2a - x)$ ...(i)

Let $I = \int_0^{2a} f(x)g(x)\,h(x)dx$

$$= \int_0^{2a} f(2a-x)g(2a-x)h(2a-x)dx$$

$$= \int_0^{2a} f(x)\cdot g(x)h(2a-x)dx$$

$$\therefore\ 3I = \int_0^{2a} f(x)g(x)3h(2a-x)\,dx \qquad \text{[Using (i)]}$$

$$= \int_0^{2a} f(x)g(x)[2h(x)-4]dx$$

$$\Rightarrow I = -4\int_0^{2a} f(x)\cdot g(x)dx \Rightarrow \frac{\int_0^{2a} f(x)g(x)h(x)dx}{\int_0^{2a} f(x)g(x)dx} = -4$$

$$\therefore \int_{-1}^{1}\left(\frac{\int_0^{2a} f(x)\,g(x)h(x)\,dx}{\int_0^{2a} f(x)\,g(x)\,dx}\right)dx = \int_{-1}^{0}(-4)dx = (-4)(2) = -8$$

11. (c) : $\int \frac{\text{cosec}^2 x - 2020}{(\cos x)^{2020}}dx$

$$= \int \underset{\text{(II)}}{\text{cosec}^2 x}\,\underset{\text{(I)}}{(\cos x)^{-2020}}\,dx - \int\frac{2020}{(\cos x)^{2020}}dx$$

$$= (\cos x)^{-2020}(-\cot x)$$

$$-\int(-2020)(\cos x)^{-2021}(-\sin x)(-\cot x)\,dx - \int\frac{2020}{(\cos x)^{2020}}dx$$

$$= -\frac{\cot x}{(\cos x)^{2020}} + \int\frac{2020}{(\cos x)^{2020}}dx - \int\frac{2020}{(\cos x)^{2020}}dx$$

$$= \frac{h(x)}{(\cos x)^{2020}} + c \cdot \text{So, } h(x) = -\cot x \quad\therefore\quad h\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

12. (b) : $\because$ $f(x) = \sin(2\sin^{-1}(\cos x))$

$$= \sin\left(2\sin^{-1}\sin\left(\frac{\pi}{2}-x\right)\right) = \sin\left(2\left(\frac{\pi}{2}-x\right)\right) = \sin 2x$$

$$\text{Now, } I = \int_0^{\pi} xf(x)\sin 2x\,dx = \int_0^{\pi} x\sin^2 2x\,dx$$

$$= \frac{\pi}{2}\int_0^{\pi}\sin^2 2x\,dx = \frac{\pi}{2}\cdot\frac{1}{2}\int_0^{2\pi}\sin^2 t\,dt = \frac{\pi}{4}\times 4\int_0^{\pi/2}\sin^2 t\,dt$$

$$= \pi\times\frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi^2}{4}$$

13. (c) : Given, $f(x) = \sin x + \int_{-\pi/2}^{\pi/2}(\sin x + t\cos x)f(t)dt$

$$= \sin x + \sin x\int_{-\pi/2}^{\pi/2} f(t)\,dt + \cos x\int_{-\pi/2}^{\pi/2} tf(t)\,dt$$

$$= \sin x\left(1 + \int_{-\pi/2}^{\pi/2} f(x)dx\right) + \cos x\int_{-\pi/2}^{\pi/2} xf(x)dx$$

$= A\sin x + B\cos x$, where

$$A = 1 + \int_{-\pi/2}^{\pi/2} f(x)\,dx \text{ and } B = \int_{-\pi/2}^{\pi/2} x\,f(x)\,dx$$

Now, $A=1+\int_{-\pi/2}^{\pi/2} f(x)dx=1+\int_{-\pi/2}^{\pi/2}(A\sin x+B\cos x)\,dx$

$=1+\int_{-\pi/2}^{\pi/2} A\sin x\,dx \;(\text{odd function}) + \int_{-\pi/2}^{\pi/2} B\cos x\,dx\;(\text{even function})$

$=1+2B\int_0^{\pi/2}\cos x\,dx \;\therefore\; A=1+2B$

Also, $B=\int_{-\pi/2}^{\pi/2} x\,f(x)dx=\int_{-\pi/2}^{\pi/2} x(A\sin x+B\cos x)\,dx$

$=A\int_{-\pi/2}^{\pi/2} x\sin x\,dx\;(\text{even function}) + B\int_{-\pi/2}^{\pi/2} x\cos x\,dx\;(\text{odd function})$

$=2A\int_0^{\pi/2} x\sin x\,dx \quad\therefore\quad B=2A=2(1+2B)$

$\Rightarrow B=-\frac{2}{3}$ and $A=-\frac{1}{3}$

So, $f(x)=-\frac{1}{3}(\sin x+2\cos x)$

$\therefore \int_0^{\pi/2} f(x)\,dx=-\frac{1}{3}\int_0^{\pi/2}(\sin x+2\cos x)dx$

$=-\frac{1}{3}\left[\left(\frac{\cos x}{-1}\right)_0^{\pi/2}+2(\sin x)_0^{\pi/2}\right]=-\frac{1}{3}(1+2)=-1$

14. (a, b, c, d) : $A(x)=\int\frac{dx}{e^{3x}+15e^{-3x}+9e^{-9x}}$

$=\int\frac{e^{9x}dx}{e^{12x}+15e^{6x}+9}$

$B(x)=\int\frac{dx}{e^{9x}+15e^{3x}+9e^{-3x}}=\int\frac{e^{3x}dx}{e^{12x}+15e^{6x}+9}$

$\therefore\quad M(x)=A(x)-3B(x)=\int\frac{e^{3x}(e^{6x}-3)}{e^{12x}+15e^{6x}+9}dx$

$=\frac{1}{3}\int\frac{t^2-3}{t^4+15t^2+9}dt$ [By putting $e^{3x}=t$]

$=\frac{1}{3}\int\frac{1-\frac{3}{t^2}}{\left(t+\frac{3}{t}\right)^2+9}dt=\frac{1}{9}\tan^{-1}\left(\frac{t+\frac{3}{t}}{3}\right)+c$

$=\frac{1}{9}\tan^{-1}\left(\frac{e^{3x}+3e^{-3x}}{3}\right)+c=\frac{1}{\lambda}\tan^{-1}(f(x))+c$

$\Rightarrow\quad \lambda=9$ and $f(x)=\frac{e^{3x}+3e^{-3x}}{3}$

$\therefore\quad f(\log 3)=\frac{e^{3\log 3}+3e^{-3\log 3}}{3}=\frac{1}{3}\left(27+\frac{1}{9}\right)=\frac{244}{27}$

$\therefore\; 27f(\log 3)=244$

and $f(\log 2)=\frac{1}{3}\left(e^{3\log 2}+3e^{-3\log 2}\right)=\frac{1}{3}\left(8+\frac{3}{8}\right)=\frac{67}{24}$

$\Rightarrow\quad 24f(\log 2)=67$

15. (a, b, c, d) : $I_{m-2,n+2}=\int\sin^{m-2}x\cos^{n+2}x\,dx$

$\Rightarrow I_{m,n}=\int\sin^m x\cos^n x\,dx$

$=-\int(\sin^{m-1}x)[\cos^n x(-\sin x)]dx$
(I) (II)

$=-\frac{\sin^{m-1}x\cos^{n+1}x}{n+1}+\frac{(m-1)}{n+1}\int\sin^{m-2}x\cos^{n+2}x\,dx$

$=-\frac{\sin^{m-1}x\cos^{n+1}x}{n+1}+\frac{m-1}{n+1}I_{m-2,n+2}$

$=-\frac{\sin^{m-1}x\cos^{x+1}x}{x+1}f(m,n)I_{m-2,n+2}$

$\therefore\quad f(m,n)=\frac{m-1}{n+1}$

$\Rightarrow f(5,4)=\frac{4}{5},\; f(4,5)=\frac{1}{2},\; f(2,2)+f(2,3)=\frac{7}{12},$

$f(2,2)+f(3,2)=\frac{1}{3}+\frac{2}{3}=1$

16. (b, c) : $g(x)=f(x+2020)$,

$f(-x)=-f(x)$, $g(-x)=g(x)$

$\therefore\quad g(-x)=f(-x+2020)$

$\Rightarrow\quad g(-x)=f-(x-2020)\Rightarrow g(x)=-f(x-2020)$

Again, $f(x+2020)=g(x)$

$\Rightarrow\quad f(t-2020+2020)=g(t-2020)$

$\Rightarrow\quad f(t)=g(2020-t)$

$\therefore\quad \int_0^{2020} f(t)dt=\int_0^{2020} g(2020-t)\,dt$

17.(a, b, c, d) : $f(x)=\frac{\sin^m x}{1+a^x}\;\Rightarrow\; f(-x)=\frac{a^x\sin^m x}{1+a^x}$

$\therefore\quad f(x)+f(-x)=\sin^m x$, which is even function as m is even.

Now, $\alpha=\int_{-\pi}^{\pi}\frac{\sin^m x}{1+a^x}dx$ and $\alpha=\int_{-\pi}^{\pi}\frac{\sin^m x a^x}{1+a^x}dx$

$\therefore 2\alpha=\int_{-\pi}^{\pi}\sin^m x\,dx=2\int_0^{\pi}\sin^m x\,dx=4\int_0^{\pi/2}\sin^m x\,dx,$

which is even function as m is even.

$$\Rightarrow \quad \alpha = 2\int_0^{\pi/2} \sin^m x\,dx$$

$$\text{For } m=4,\ \alpha = 2\int_0^{\pi/2} \sin^4 x dx = 2\cdot\frac{3}{4}\cdot\frac{1}{2}\times\frac{\pi}{2} = \frac{3\pi}{8}$$

which is independent of a.

$$\text{For } m=2,\ \alpha = 2\cdot\int_0^{\pi/2} \sin^2 x dx = 2\times\frac{1}{2}\times\frac{\pi}{2} = \frac{\pi}{2}$$

18. (b, d) : The given ellipse is $\frac{x^2}{36}+\frac{y^2}{25}=1$

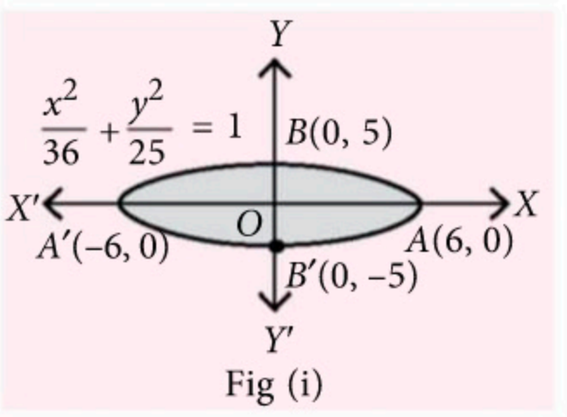


Fig (i)

$$\therefore \quad \text{Area of figure (i)} = 4\int_0^6 y dx = 4\times 5\int_0^6 \sqrt{\left(1-\frac{x^2}{36}\right)}dx$$

$$= \frac{20}{6}\int_0^6 \sqrt{6^2-x^2}\,dx = \frac{20}{6}\left[\frac{x}{2}\sqrt{6^2-x^2}+\frac{36}{2}\left(\sin^{-1}\frac{x}{6}\right)\right]_0^6$$

$$= \frac{20}{6}\times 18\times\frac{\pi}{2} = 30\pi$$

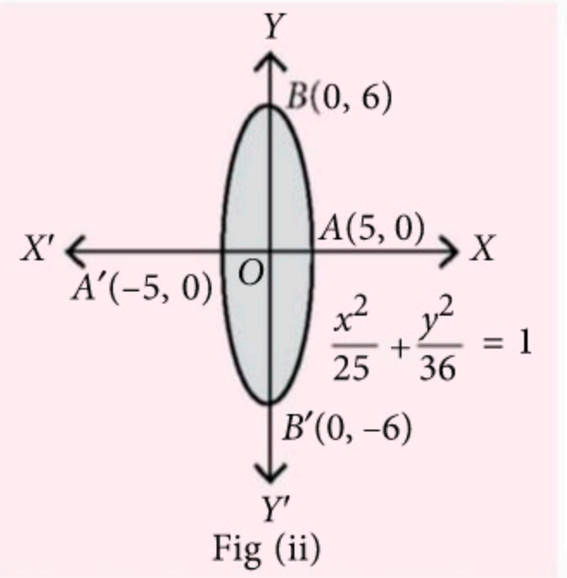


Fig (ii)

$$\text{Area of figure (ii)} = 4\times\int_0^5 y dx$$

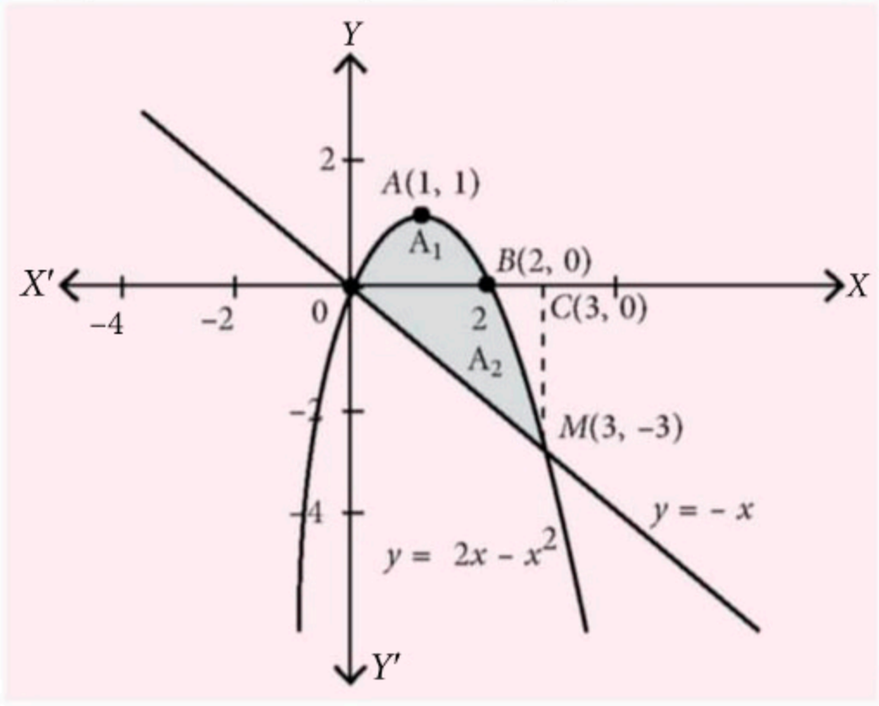
$$= 4\times 6\int_0^5 \sqrt{1-\frac{x^2}{25}}dx = \frac{24}{5}\int_0^5 \sqrt{5^2-x^2}\,dx$$

$$= \frac{24}{5}\left[\frac{x}{2}\sqrt{5^2-x^2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5 = \frac{24}{5}\times\frac{25}{2}\times\frac{\pi}{2} = 30\pi$$

$\therefore$ Area bounded by the curves $\frac{x^2}{36}+\frac{y^2}{25}=1$ and $\frac{x^2}{25}+\frac{y^2}{36}=1 (a<b)$ are same.

19. (a, c, d) : The given curve is $y = 2x - x^2$ which is downward parabola with vertex $A(1, 1)$ and intersecting x-axis at $O(0, 0)$ and $B(2, 0)$.

Solving $y = 2x - x^2$ and $y = -x$, we get $O(0, 0)$, $M(3, -3)$



$$\therefore \quad A_1 = \int_0^2 (2x-x^2)dx = \frac{4}{3}\text{ sq.units}$$

$$\text{and } A_2 = -\left[\int_0^3 (-x)dx - \int_2^3 (2x-x^2)dx\right]$$

[$\because$ A_2 is lying below the x-axis]

$$= -\left[\left(-\frac{x^2}{2}\right)_0^3 - \left(x^2-\frac{x^3}{3}\right)_2^3\right] = \frac{19}{6}\text{ sq. units}$$

$$\therefore\ A_1 + A_2 = \frac{4}{3}+\frac{19}{6} = \frac{9}{2}\text{ sq. units}$$

Also, $A_2 > A_1$ and $\left|\frac{A_1}{A_2}\right| = \frac{8}{19}$

20. (a, b, c, d) : $\int \frac{x^x(x^{4x}+1)(1+\log x)}{1+x^{6x}}dx$

$$= \int \frac{t^4+1}{1+t^6}dt \quad [\text{By putting } x^x = t]$$

$$= \int \frac{(t^4-t^2+1)+t^2}{(t^2+1)(t^4-t^2+1)}dt = \int \frac{dt}{1+t^2}+\int \frac{t^2}{1+(t^3)^2}dt$$

$$= \tan^{-1} t + \frac{1}{3}\tan^{-1} t^3 = \tan^{-1} x^x + \frac{1}{3}\tan^{-1} x^{3x} + c$$

$$= \sum_{r=1}^{3} a_r \tan^{-1} x^{rx} + c$$

$= a_1 \tan^{-1} x^x + a_2 \tan^{-1} x^{2x} + a_3 \tan^{-1} x^{3x} + c$

$\therefore \quad a_1 = 1, a_2 = 0, a_3 = \frac{1}{3}$

$\therefore \quad a_1 \cdot a_2 \cdot a_3 = 0$ and $a_2 < a_3 < a_1$

Now, $\sum_{r=1}^{3} r a_r = 1 \cdot a_1 + 2 \cdot a_2 + 3 \cdot a_3$

$= 1 + 0 + 3\left(\frac{1}{3}\right) = 2$ and $a_1 a_3 = 1 \cdot \frac{1}{3} > 0$

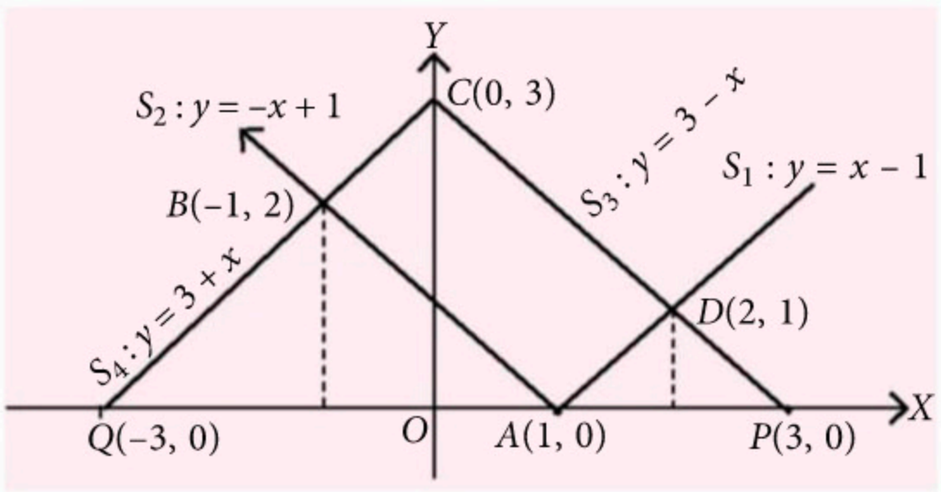
(21-24) : The curve S_1, S_2, S_3 and S_4 meet x-axis at $A(1, 0)$, $P(3, 0)$ and $Q(-3, 0)$

The curves S_1 and S_3 intersect at $D(2, 1)$.

The curves S_2 and S_4 intersect at $B(-1, 2)$.

The curves S_3 and S_4 intersect at $C(0, 3)$.

The curves S_1 and S_2 intersect at x-axis at $A(1, 0)$.



21. (a) : Area bounded by curves S_1, S_3 with x-axis

$= \int_1^2 S_1 dx + \int_2^3 S_3 dx = \int_1^2 (x-1)dx + \int_2^3 (3-x)dx$

$= \frac{1}{2}\left[(x-1)^2\right]_1^2 - \left[\frac{1}{2}(3-x)^2\right]_2^3 = \frac{1}{2}(1-0) - \frac{1}{2}(0-1) = 1$

22. (b) : Area bounded by the curves S_2, S_4 with x-axis

$= \int_{-3}^{-1} S_4 dx + \int_{-1}^{1} S_2 dx = \int_{-3}^{-1} (3+x)dx + \int_{-1}^{1} (-x+1)dx$

$= \left[\frac{1}{2}(3+x)^2\right]_{-3}^{-1} - \left[\frac{1}{2}(-x+1)^2\right]_{-1}^{1} = 2 + 2 = 4$

23. (d) : Area bounded by the curves S_3, S_4 with x-axis

$= \int_{-3}^{0} S_4 dx + \int_{0}^{3} S_3 dx = \int_{-3}^{0} (3+x)dx + \int_{0}^{3} (3-x)dx = 9$

24. (c) : Area bounded by curves S_1, S_2, S_3 and S_4 is = Area bounded by curves S_3 and S_4 with x-axis – (Area bounded by curves S_1 and S_3 with x-axis + Area bounded by curves S_2 and S_4 with x-axis)

$= 9 - (1 + 4) = 9 - 5 = 4$

25. **P → (2); Q → (5); R → (4); S → (1)**

(P) $\int_0^{\pi} \sin mx \sin nx\, dx = \int_0^{\pi} \sin^2 mx\, dx \quad (\because m = n)$

$= \frac{1}{m}\int_0^{m\pi} \sin^2 t\, dt = \int_0^{\pi} \sin^2 x dx$

$= 2\int_0^{\pi/2} \sin^2 x dx = 2 \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{2}$

(Q) Let $I = \int_{\pi/6}^{\pi/3} \frac{\sin^{2020} x}{\cos^{2020} x + \sin^{2020} x} dx$ ...(i)

or $I = \int_{\pi/6}^{\pi/3} \frac{\cos^{2020} x}{\sin^{2020} x + \cos^{2020} x} dx$ ...(ii)

$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx$ [On adding (i) and (ii)]

$\Rightarrow \quad 2I = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) \quad \Rightarrow I = \frac{\pi}{12} = \frac{k}{4} \quad \therefore \quad k = \frac{\pi}{3}$

(R) $\int_0^{\infty} \frac{\sin^3 x}{x} dx = \frac{1}{4}\int_0^{\infty} \frac{3\sin x - \sin 3x}{x} dx$

$= \frac{3}{4}\int_0^{\infty} \frac{\sin x dx}{x} - \frac{1}{4}\int_0^{\infty} \frac{\sin 3x}{x} dx$

$= \frac{3}{4} \times \frac{\pi}{2} - \frac{1}{4}\int_0^{\infty} \frac{\sin t}{t} dt = \frac{3}{4} \times \frac{\pi}{2} - \frac{1}{4}\int_0^{\infty} \frac{\sin x}{x} dx$

$= \frac{3\pi}{8} - \frac{1}{4} \times \frac{\pi}{2} = \frac{2\pi}{8} = \frac{\pi}{4}$

(S) Given, $k = \int_0^{\pi/2} \sin 2x \log(\tan x) dx$ ...(i)

$= \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2} - x\right) \log \tan\left(\frac{\pi}{2} - x\right) dx$

$\therefore k = \int_0^{\pi/2} \sin 2x \log(\cot x) dx$ ...(ii)

Adding (i) and (ii), we get

$2k = \int_0^{\pi/2} \sin 2x [\log(\tan x) + \log(\cot x)] dx$

$= \int_0^{\pi/2} \sin 2x\, (\log 1)\, dx \Rightarrow k = 0$

26. (256) : $\int \frac{4x^3 + 3x^2}{x^8 + 2x^7 + x^6 + 4x^4 + 4x^3 + 13} dx$

$$= \int \frac{4x^3+3x^2}{(x^4+x^3)^2+4(x^4+x^3)+13}\,dx$$

$$= \int \frac{d(x^4+x^3)}{(x^4+x^3)^2+4(x^4+x^3)+13}\,dx$$

$$= \int \frac{dt}{t^2+4t+13} \qquad [\text{Putting } t = x^4+x^3]$$

$$= \int \frac{dt}{(t+2)^2+3^2} = \frac{1}{3}\tan^{-1}\left(\frac{t+2}{3}\right)+D$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x^4+x^3+2}{3}\right)+D$$

$$= \frac{\alpha}{\beta}\tan^{-1}(Ax^4+Bx^3+C)+D \qquad [\text{Given}]$$

$$\Rightarrow \alpha = 1, \beta = 3, A = \frac{1}{3}, B = \frac{1}{3}, C = \frac{2}{3}$$

$$\therefore \quad (\alpha+\beta)^4 \cdot \left(\frac{A+B}{C}\right)^4 = 4^4 = 256$$

27. (18) : Given $f(x) = x^2 + \int_0^x e^{-t} f(x-t)\,dt$ ...(i)

Putting $x - t = z \Rightarrow dt = -dz$. So, $e^{-t} = e^z \cdot e^{-x}$

At $t = 0$, $z = x$ and at $t = x$, $z = 0$

$$\therefore \quad f(x) = x^2 + \int_x^0 \frac{e^z e^{-x} f(z)}{-1}\,dz = x^2 + e^{-x}\int_0^x e^z f(z)\,dz$$

$$\Rightarrow f(x) = x^2 + e^{-x}\int_0^x e^t f(t)\,dt \qquad ...(ii)$$

$$\therefore \quad f'(x) = 2x + e^{-x}\cdot e^x f(x) + \frac{e^{-x}}{-1}\int_0^x e^t f(t)\,dt$$

$$\Rightarrow \quad f'(x) = 2x + f(x) - [f(x) - x^2] \qquad [\text{Using (ii)}]$$

$$\Rightarrow \quad f'(x) = 2x + x^2 \quad \Rightarrow \quad f(x) = x^2 + \frac{x^3}{3} + c$$

From (i), we have $f(0) = 0 \Rightarrow \quad c = 0$

So, $\quad f(x) = \frac{x^3}{3} + x^2 \quad \Rightarrow \quad f(3) = 9+9 = 18$

28. (2020) : Let $I = \int_0^1 \underset{\text{I}}{(1-x^5)^{404}} \cdot \underset{\text{II}}{1}\,dx$

$$= \left[x(1-x^5)^{404}\right]_0^1 - 404\times 5\int_0^1 (-x^4)(1-x^5)^{403}\cdot x\,dx$$

$$= -2020\int_0^1 (1-x^5)^{403}(1-x^5-1)\,dx$$

$$= -2020\int_0^1 (1-x^5)^{404}\,dx + 2020\int_0^1 (1-x^5)^{403}\,dx$$

$$\therefore \quad 2021I = 2020\int_0^1 (1-x^5)^{403}\,dx$$

$$\Rightarrow \quad \frac{\int_0^1 (1-x^5)^{404}\,dx}{\int_0^1 (1-x^5)^{403}\,dx} = \frac{2020}{2021} = \lambda \quad \therefore \quad 2021\lambda = 2020$$

29. (3) : Given, $A_1 = \int_0^{3\pi} f(\cos^2 x)\,dx$

$$\therefore \quad A_1 = 3\int_0^{\pi} f(\cos^2 x)\,dx = 3A_2 \quad \Rightarrow \quad \frac{A_1}{A_2} = 3$$

30. (8) : Let $I = \int_{-2}^{2} \frac{3x^2}{1+e^x}\,dx$ ... (i)

Put $x = -t \Rightarrow dx = -dt$

Now at $x = 2$, $t = -2$ and at $x = -2$, $t = 2$

$$\therefore \quad I = -\int_2^{-2} \frac{3t^2}{1+e^{-t}}\,dt = \int_{-2}^{2} \frac{3t^2e^t}{1+e^t}\,dt = \int_{-2}^{2} \frac{3x^2e^x}{1+e^x}\,dx \qquad ... (ii)$$

Adding (i) and (ii), we get

$$2I = \int_{-2}^{2} 3x^2\,dx = 2\left[x^3\right]_0^2 \Rightarrow \quad I = 8$$

JEE Main April - 2020

Schedule of Events with Dates

Events	Dates
On-line submission of Application Form	07.02.2020 to 06.03.2020 (Upto 11:50 p.m.)
Dates of Examination	Between 05.04.2020, 07.04.2020 to 09.04.2020 & 11.04.2020
Timing of Examinations	First Shift : 9:30 a.m. to 12:30 p.m.
	Second Shift : 2:30 p.m. to 5:30 p.m.
Declaration of Result on NTA's website	By 30.04.2020

Examination Pattern

	Subject	No. of Questions	Marks
Subject wise Distribution of Questions, Total Number of Questions and Marks	Mathematics	(20+5*)	100
	Physics	(20+5*)	100
	Chemistry	(20+5*)	100
	Total	75	300
*20 Questions will be MCQs and 5 Questions will have answer to be filled in as Numerical value.			
Marking Scheme for MCQs	Correct Answer or the best Answer	Four mark (+4)	
	Incorrect Answer	Minus one mark (–1)	
	Unanswered / Marked for Review	No mark (0)	
Marking Scheme for Questions for which answer is a Numerical value	Correct Answer	Four mark (+4)	
	Incorrect Answer	No mark (0)	
	Unanswered / Marked for Review	No mark (0)	

For detailed information refer to www.jeemain.nta.nic.in

JEE 2020 ADVANCED

Exam on 17th May

PRACTICE PAPER

* ALOK KUMAR

Single Option Correct Type

1. If the range of $f(x)=\begin{cases}2+\sqrt[3]{x}, -3\le x<-1\\ x^{2/3}, -1\le x\le 2\end{cases}$ is $[0,\sqrt[3]{n}]$, where $n\in N$, then $n=$

(a) 1 (b) 2 (c) 4 (d) 6

2. If A, B, C are angles in a ΔABC and $\sin\left(A-\frac{\pi}{4}\right)\sin\left(B-\frac{\pi}{4}\right)\sin\left(C-\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}$, then $\Sigma \tan A\tan B=$

(a) $\Sigma\tan A$ (b) $\Sigma\cot A$

(c) $(\Sigma\tan A)^{-1}$ (d) $(\Sigma\cot A)^{-1}$

3. The probability that a randomly chosen positive divisor of 10^{99} is an integer multiple of 10^{88} is

(a) $\left(\frac{3}{25}\right)^2$ (b) $\left(\frac{3}{5}\right)^2$ (c) $\left(\frac{8}{9}\right)^2$ (d) $\left(\frac{8}{9}\right)^4$

4. The number of ordered pairs of positive integers (a, b) such that L.C.M. of a and b is $2^3 5^7 11^{13}$ is

(a) 2385 (b) 2835 (c) 3825 (d) 8325

5. A license plate consists of 8 digits out of 10 digits 0, 1, 2, 3, ..., 9.It is called even if it contain an even number of 0's.The number of even license plates is

(a) $\frac{10^8-8^8}{2}$ (b) 10^8-8^{10}

(c) $\frac{10^8+8^8}{2}$ (d) $\frac{10^8+8^{10}}{2}$

6. The shape of surface of a curved mirror such that light from a source at origin will be reflected in a beam of rays parallel to x-axis is

(a) circle (b) parabola

(c) ellipse (d) hyperbola

7. The area of the loop of the curve $y^2=x^4(x+2)$ is [in sq. units]

(a) $\frac{32\sqrt{2}}{105}$ (b) $\frac{64\sqrt{2}}{105}$ (c) $\frac{128\sqrt{2}}{105}$ (d) $\frac{256\sqrt{2}}{105}$

8. If $n\in N$, then the remainder when $37^{n+2}+16^{n+1}+30^n$ is divided by 7 is

(a) 0 (b) 1 (c) 2 (d) 5

9. If a, b, c are three complex numbers such that $a^2+b^2+c^2=0$ and $\Delta=\begin{vmatrix} b^2+c^2 & ab & ac\\ ab & c^2+a^2 & bc\\ ac & bc & a^2+b^2\end{vmatrix}=ka^2b^2c^2$, then the value of k is

(a) 1 (b) 2 (c) −2 (d) 4

10. The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vectors $\vec{a}, \vec{b}, \vec{c}$ such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=1/2$. Then the volume of the parallelepiped is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $3\sqrt{2}$ (d) $\frac{1}{\sqrt{3}}$

11. If an ellipse slides between two perpendicular straight lines, then the locus of its centre is

(a) a parabola (b) an ellipse

(c) a hyperbola (d) a circle

12. Let P and Q be the respective intersections of the internal and external angle bisectors of the triangle ABC at C and the side AB produced. If $CP = CQ$, then the value of (a^2+b^2) is (where a and b and R have their usual meanings for ΔABC)

(a) $2R^2$ (b) $2\sqrt{2}R^2$ (c) $4R^2$ (d) $4\sqrt{2}R^2$

13. The value of the definite integral $\int_{-1}^{1}\frac{dx}{(1+e^x)(1+x^2)}$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/8$ (d) $\pi/16$

*Alok Kumar, a B Tech from IIT Kanpur and INMO 4th ranker of his time, has been training IIT and Olympiad aspirants for close to two decades now. His students have bagged AIR 1 in IIT JEE and also won medals for the country at IMO. He has also taught at Maths Olympiad programme at Cornell University, USA and UT, Dallas. He has been regularly proposing problems in international Mathematics journals.

14. For a curve $y=f(x), f''(x)=4x$ at each point (x, y) on it and it crosses the x-axis at $(-2, 0)$ an angle of $45°$ with positive direction of x-axis. If $f(x) = ax^3 + bx^2 + cx + d$, then the value of $3(a + b + c + d)$ equals

(a) 45 (b) −15 (c) −45 (d) 15

15. The value of the definite integral $\int_0^{n\pi} \frac{x|\sin x|}{1+|\cos x|}dx$ $(n \in N)$ is equal to

(a) $n^2\pi\ln 2$ (b) $n\pi^2\ln 2$ (c) $n\pi\ln 2$ (d) $n^2\pi\ln 2/2$

16. The equation $\sin x + x\cos x = 0$ has at least one real root in

(a) $\left(-\frac{\pi}{2}, 0\right)$ (b) $(0, \pi)$

(c) $\left(\pi, \frac{3\pi}{2}\right)$ (d) $\left(0, \frac{\pi}{2}\right)$

One or More Than One Option(s) Correct Type

17. If $a=\lim_{n\to\infty}\left(\frac{(2n)!}{n!n^n}\right)^{\frac{1}{n}}$ and

$b=\lim_{n\to\infty}\left(\left(1+\frac{1}{n^2}\right)^{\frac{2}{n^2}}\left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}}\ldots\ldots 2^{\frac{2}{n}}\right)$, then

(a) $a = b$ (b) $a=\frac{4}{e}$

(c) $b=\frac{2}{e}$ (d) $a=b=\frac{4}{e}$

18. The number of isosceles triangles with integer sides if no side exceeds 2008 is

(a) $(1004)^2$ if equal sides do not exceed 1004

(b) $2(1004)^2$ if equal sides exceed 1004

(c) $3(1004)^2$ if equal sides have any length ≤ 2008

(d) $(2008)^2$ if equal sides have any length ≤ 2008

19. If the tangents of the angles A and B of a triangle ABC satisfy the equation $abx^2 - c^2x + ab = 0$, then

(a) $\tan A = a/b$ (b) $\tan B = b/a$

(c) $\cos C = 0$

(d) $\sin^2 A + \sin^2 B + \sin^2 C = 2$

20. Which of the following statement(s) is always true?

(a) The function $f(x) = \ln(x^2 + 1) - e^{-x} + 1$ is strictly increasing $\forall\ x \in R$.

(b) $\lim_{x\to 0^+} x\ln x(1+\ln x)$ does not exist.

(c) Least value of the function $f(x)=\frac{x^4+x^2+5}{(x^2+1)^2}$ is one.

(d) A line which is perpendicular to a plane is perpendicular to every line in the plane.

21. Let $L=\lim_{n\to\infty}\int_a^{\infty}\frac{ndx}{1+n^2x^2}$, where $a \in R$, then L can be

(a) π (b) $\pi/2$ (c) 0 (d) 1

22. Consider the function,

$f(x) = x^3 - 9x^2 + 15x + 6$ for $1 \le x \le 6$ and

$g(x)=\begin{cases}\min.f(t) & \text{for } 1\le t\le x, 1\le x\le 6\\ x-18 & \text{for } x>6\end{cases}$, then which of the following hold(s) good ?

(a) $g(x)$ is differentiable at $x = 2$

(b) $g(x)$ is discontinuous at $x = 6$

(c) $g(x)$ is continuous and derivable at $x = 5$

(d) $g(x)$ is monotonic in $(1, 5)$

23. If $\vec{a}, \vec{b}, \vec{c}$ be three non zero vectors satisfying the condition $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$, then which of the following always hold(s) good ?

(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs

(b) $[\vec{a}\ \vec{b}\ \vec{c}]=|\vec{b}|$ (c) $[\vec{a}\ \vec{b}\ \vec{c}]=|\vec{c}|^2$

(d) $|\vec{a}|=|\vec{c}|$

Comprehension Type

Paragraph for Q. No. 24 to 26

If A is 3×3 matrix, then a non trivial solution $X = (x\ y\ z)^T$ such that $AX = \lambda X(\lambda \in R)$ yeilds 3 values of λ say $\lambda_1, \lambda_2, \lambda_3$. For any such matrix A, λ' s are called eigen values and corresponding X's are called eigen vectors. It is known that ,for any 3×3 matrix, $Tr(A) = \lambda_1 + \lambda_2 + \lambda_3$, $\det A = \lambda_1\lambda_2\lambda_3$. Answer the following questions for matrix, $A=\begin{bmatrix}1 & 1 & 2\\ 2 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}$.

24. $Tr(A^{-1}) =$

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{3}$

25. $Tr(A^3) =$

(a) 149 (b) 101 (c) 128 (d) 133

26. Which of the following is false?

(a) $\exists$ nontrivial solution X such that $AX=(2+\sqrt{7})X$

(b) $\exists$ a nontrivial solution X such that $AX = X$

(c) $\exists$ a nontrivial solution X such that $A^{-1}X=(2-\sqrt{7})X$

(d) The total number of nontrivial solutions X such that $AX = \lambda X$ is 3.

Paragraph for Q. No. 27 to 29

Starting at (0, 0) an object moves in xy-plane via a sequence of steps, each of length 1 unit. Each step is left, right, up or down ,all the four being equally likely. The probability that object reaches (2, 2) in

27. Exactly 4 steps is

(a) $\frac{5}{128}$ (b) $\frac{3}{128}$ (c) $\frac{1}{128}$ (d) $\frac{1}{256}$

28. Exactly 6 steps is

(a) $\frac{6}{4^4}$ (b) $\frac{1}{4^6}$ (c) $\frac{6}{4^6}$ (d) $\frac{15}{4^4}$

29. Six or fewer steps is

(a) 1/16 (b) 1/32 (c) 3/64 (d) 5/64

Matrix-Match Type

30. Match the following :

Column-I		Column-II	
(A)	If $\log\left(\frac{x^2}{y^3}\right)=1$ and $\log(x^2y^3)=7$, then $\log(\lvert xy\rvert)$ is equal to	(p)	0
(B)	If $f(x)=\min.\{\lvert x\rvert, x^2, 2\}$, $x\in[-5,5]$, then number of points where $f(x)$ is not derivable, is	(q)	3/2
(C)	$\int_0^2 \frac{2x^3-6x^2+9x-5}{x^2-2x+5}dx=$	(r)	3
(D)	If the range of the function $f(x)=\log_2(4^{x^2}+4^{(x-1)^2})$ is $[a,\infty)$, then the value of 'a' equals	(s)	4

SOLUTIONS

1. (c) : The given function has local maximum at $x = -1$ and local minimum at $x = 0$.

Also $f(0) = 0, f(-1) = 1, f(-3)=2-\sqrt[3]{3}$ and $f(2)=2^{2/3}=\sqrt[3]{4}$

$\therefore$ Range of $f(x) = [0, \sqrt[3]{4}]$

2. (a) : Given equation can be written as

$(\sin A - \cos A)(\sin B - \cos B)(\sin C - \cos C) = 1$

$\Rightarrow (\tan A - 1)(\tan B - 1)(\tan C - 1) = \sec A \sec B \sec C$

$\Rightarrow (\Sigma\tan A - \Sigma\tan A \tan B + \Sigma\tan A - 1)$
$= \sec A \sec B \sec C$

$\Rightarrow 2\,\Sigma\tan A - \Sigma\tan A \tan B = 1 + \sec A \sec B \sec C$

Also, in ΔABC if $A + B + C = \pi$, then

$\Sigma\tan A \tan B = 1 + \sec A \sec B \sec C$

$\therefore$ $2\,\Sigma\tan A = 2\Sigma \tan A \tan B \Rightarrow \Sigma\tan A \tan B = \Sigma\tan A$

3. (a) : $10^{99} = 2^{99}\cdot5^{99}$ has totally $(100)^2$ divisors of the form $2^a \cdot 5^b$ $(0 \le a, b \le 99)$. Of these multiples of 10^{88} are of the form $2^a \cdot 5^b$ $(88 \le a, b \le 99)$. So, there are 12^2 choices for both a and b.

$\therefore$ Required probability $= \frac{12^2}{100^2}=\left(\frac{3}{25}\right)^2$

4. (b) : a, b are factors of the form $2^{a_1}5^{b_1}11^{c_1}$, $2^{a_2}5^{b_2}11^{c_2}$, where $a_1, b_1, c_1, a_2, b_2, c_2$ are non-negative integers. Since, L.C.M. of a, b is $2^3\,5^7\,11^{13}$

$\therefore$ Max $\{a_1, a_2\} = 3$, max $\{b_1, b_2\} = 7$ and max $\{c_1, c_2\} = 13$.

Hence (a_1, a_2) can be (0, 3), (1, 3), (2, 3), (3, 3), (3, 2), (3, 1), (3, 0) and giving us 7 choices.Similarly (b_1, b_2) has 15 choices and (c_1, c_2) has 27 choices.

Hence, total number of choices $= 7 \times 15 \times 27 = 2835$

5. (c) : For an even license plate, there must be $2K$ zeros $(0 \le K \le 4)$ and $8 - 2K$ non zero digits, each of which has 9 choices. There are ${}^8C_{2K}$ ways to choose $2K$ places for 0's and $({}^8C_{2K})\ 9^{8-2K}$ plates have exactly $2K$ zeroes. Therefore the required answer is

$$\sum_{K=0}^{4}({}^8C_{2K})\,9^{8-2K} = {}^8C_0 9^8 + {}^8C_2 9^6 + {}^8C_4 9^4 + {}^8C_6 9^2 + {}^8C_8$$
$$= \frac{(9+1)^8+(9-1)^8}{2} = \frac{10^8+8^8}{2}$$

6. (b)

7. (d) : Area $= 2\int_{-2}^{0} y\,dx = 2\int_{-2}^{0} x^2\sqrt{x+2}\,dx$

$$= 4\int_0^{\sqrt{2}} (z^2-2)^2 z^2\,dz \quad \left[\text{Putting } \sqrt{x+2}=z\right]$$

$$= 4\left[\frac{z^7}{7}-\frac{4z^5}{5}+\frac{4z^3}{3}\right]_0^{\sqrt{2}} = \frac{256\sqrt{2}}{105}$$

8. (a) : $37^{n+2} = (5 \times 7 + 2)^{n+2}$
= a multiple of $7 + 2^{n+2}$

$16^{n+1} = (2 \times 7 + 2)^{n+1}$ = a multiple of $7 + 2^{n+1}$

and $30^n = (4 \times 7 + 2)^n$ = a multiple of $7 + 2^n$

9. (d) : $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

$$= abc\begin{vmatrix} -a & a & a \\ b & -b & b \\ c & c & -c \end{vmatrix} = a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= a^2b^2c^2\begin{vmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 4a^2b^2c^2 \quad \therefore \quad K = 4$$

Monthly Test Drive - 10 CLASS XI **ANSWER KEY**

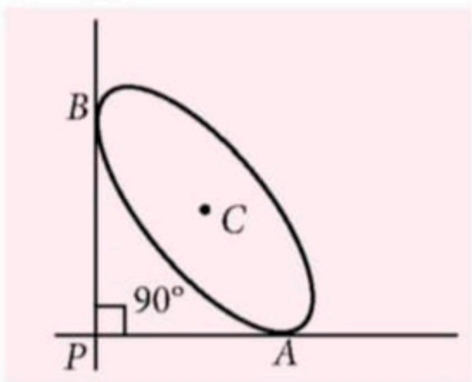
1.	(b)	**2.**	(c)	**3.**	(b)	**4.**	(d)
5.	(d)	**6.**	(d)	**7.**	(a, c)	**8.**	(c, d)
9.	(a, c)	**10.**	(a, c)	**11.**	(b)	**12.**	(a,b,c,d)
13.	(a,b,c)	**14.**	(b)	**15.**	(a)	**16.**	(c)
17.	(74)	**18.**	(0.28)	**19.**	(1)	**20.**	(1.5)

10. (a) : (Volume)^2 $= [\vec{a}\ \vec{b}\ \vec{c}]\,[\vec{a}\ \vec{b}\ \vec{c}]$

$$= \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{b}\cdot\vec{a} & \vec{c}\cdot\vec{a} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{b} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2}$$

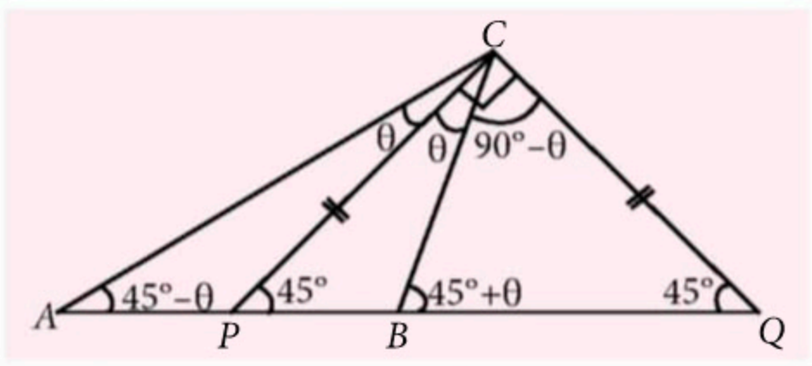
$\therefore$ Volume $= \dfrac{1}{\sqrt{2}}$.

11. (d) : Let $2a$, $2b$ be the length of the major and minor axes respectively of the ellipse. If the ellipse slides between two perpendicular lines, the point of intersection P of these lines being the point of intersection of perpendicular tangents lies on the director circle of the ellipse.



$\Rightarrow$ Center C of ellipse is always at a distance of $\sqrt{a^2+b^2}$ from P.

12. (c) :



$a^2 + b^2 = 4R^2(\sin^2(45° - \theta) + \sin^2(135° - \theta))$
$= 4R^2(\sin^2(45° - \theta) + \cos^2(45° - \theta)) = 4R^2$

13. (b) : Let $I = \displaystyle\int_{-1}^{1} \frac{dx}{(1+e^x)(1+x^2)}$ ...(i)

$\Rightarrow \quad I = \displaystyle\int_{-1}^{1} \frac{dx}{(1+e^{-x})(1+x^2)}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_{-1}^{1} \frac{dx}{1+x^2} = 2\int_{0}^{1} \frac{dx}{1+x^2} \Rightarrow I = \left[\tan^{-1} x\right]_0^1 = \pi/4$$

14. (c) : Given, $f''(x) = 4x \Rightarrow f'(x) = 2x^2 + c_1$

$\Rightarrow \quad 1 = 8 + c_1$

[$\because$ Curve cuts x-axis at $(-2, 0)$ and slope is tan 45°]

$\Rightarrow \quad c_1 = -7$

$\therefore \quad f'(x) = 2x^2 - 7 \Rightarrow f(x) = \dfrac{2x^3}{3} - 7x + c_2$

$\Rightarrow \quad 0 = \dfrac{-16}{3} + 14 + c_2$ [$\because$ Curve passes through $(-2, 0)$]

$\Rightarrow \quad c_2 = \dfrac{-26}{3}$

$\therefore \quad f(x) = \dfrac{2x^3}{3} - 7x - \dfrac{26}{3}$

Also, $f(x) = ax^3 + bx^2 + cx + d$

$\therefore \quad a = \dfrac{2}{3},\ b = 0,\ c = -7,\ d = -\dfrac{26}{3}$

So, $3(a+b+c+d) = 3\left(\dfrac{2}{3} + 0 - 7 - \dfrac{26}{3}\right) = -45$

15. (a) : $I = \displaystyle\int_0^{n\pi} \frac{x|\sin x|}{1+|\cos x|}\,dx$

$$\Rightarrow \quad 2I = n\pi \int_0^{n\pi} \frac{|\sin x|}{1+|\cos x|}\,dx = n^2\pi \int_0^{\pi} \frac{\sin x}{1+|\cos x|}\,dx$$

$$= 2n^2\pi \cdot \int_0^{\pi/2} \frac{\sin x}{1+\cos x}\,dx \quad \therefore\ I = n^2\pi \ln 2$$

16. (b) : Let $g(x) = \displaystyle\int_0^x (\sin x + x\cos x)\,dx$

$g(0) = 0 = g(\pi)$

By Rolle's theorem, $\exists$ at least one real root $\in (0, \pi)$ such that $g'(x) = 0$, *i.e.*, $\sin x + x\cos x = 0$

17. (a, b, d) : $\ln a = \displaystyle\lim_{n\to\infty} \sum_{r=0}^{n-1} \frac{1}{n} \ln\left(2 - \frac{r}{n}\right)$

$$= \int_0^1 \ln(2-x)\,dx = \ln 4 - 1 \quad \Rightarrow \quad a = \frac{4}{e}$$

$$\ln b = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2r}{n^2} \ln\left(1 + \left(\frac{r}{n}\right)^2\right) = \int_0^1 2x \ln(1+x^2)\,dx$$

$\Rightarrow \quad \ln b = \ln\dfrac{4}{e} \Rightarrow b = \dfrac{4}{e}$

18. (a, b, c) : If the sides are a, a, b, then the triangle forms only when $2a > b$. So for any $a \in N$, b can change from 1 to $2a - 1$, where $a \le 1004$

$\Rightarrow$ Number of triangles $= 1 + 3 + 5 + \ldots + (2(1004) - 1)$
$= (1004)^2$

And if $1005 \le a \le 2008$, b can take any value from 1 to 2008. But a has 1004 possibilities.

$\therefore$ Number of triangles $= 1004 \times 2008 = 2(1004)^2$

$\therefore$ Total number of isosceles triangles $= 3(1004)^2$

19. (a, b, c, d) : $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

20. (a, d)

21. (a,b,c) : $L = \displaystyle\lim_{n\to\infty} \int_a^{\infty} \frac{n\,dx}{1+n^2x^2} = \lim_{n\to\infty} n \cdot \left[\frac{\tan^{-1}(nx)}{n}\right]_a^{\infty}$

$$= \lim_{n\to\infty} \left(\frac{\pi}{2} - \tan^{-1}(an)\right) = \begin{cases} \pi, & a < 0 \\ \dfrac{\pi}{2}, & a = 0 \\ 0, & a > 1 \end{cases}$$

22. (a, b, c, d) : Let $f(t) = t^3 - 9t^2 + 15t + 6$

$f'(t) = 3t^2 - 18t + 15 = 3(t - 1)(t - 5)$

$$g(x)=\begin{cases} x^3-9x^2+15x+6, & 1\le x<5 \\ -19, & 5\le x\le 6 \\ x-18 & x>6 \end{cases}$$

From above one can observe all.

23. (a, c, d) : $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow$ $\vec{a}, b, \vec{c}$ are orthogonal to each other.

Also, $\vec{a}\times\vec{b}=\vec{c} \Rightarrow |\vec{a}||\vec{b}|=|\vec{c}|$ and $\vec{b}\times\vec{c}=\vec{a}$

$\Rightarrow |\vec{b}||\vec{c}|=|\vec{a}| \therefore |\vec{b}|=1$ and $|\vec{a}|=|\vec{c}|$

Now, $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})=|\vec{a}||\vec{b}||\vec{c}|=|\vec{c}|^2$

(24-26) :

24. (d) **25. (b)** **26. (c)**

For the matrix $A=\begin{bmatrix}1 & 1 & 2\\ 2 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}$, we have

$|A - \lambda I| = 0 \Rightarrow \lambda^3 - 5\lambda^2 + \lambda + 3 = 0$ ... (i)

whose roots are $\lambda_1, \lambda_2, \lambda_3$

$\lambda_1 + \lambda_2 + \lambda_3 = 5, \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 1, \lambda_1\lambda_2\lambda_3 = -3$

$\Rightarrow$ $Tr(A) = 5$, $\det(A) = -3$

Also, $AX=\lambda X \Rightarrow A^{-1}X=\frac{1}{\lambda}X$

$$\Rightarrow Tr(A^{-1})=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3}=\frac{\sum\lambda_1\lambda_2}{\lambda_1\lambda_2\lambda_3}=-\frac{1}{3}$$

Now, $AX=\lambda X \Rightarrow A^3X=\lambda^3X \Rightarrow Tr(A^3)=\lambda_1^3+\lambda_2^3+\lambda_3^3$

$=(\lambda_1+\lambda_2+\lambda_3)(\lambda_1^2+\lambda_2^2+\lambda_3^2-\lambda_1\lambda_2-\lambda_2\lambda_3-\lambda_3\lambda_1)$

$+ 3\lambda_1\lambda_2\lambda_3$

$= 5(25 - 3(1)) - 9 = 101$

Solving (i) gives $\lambda_1=1, \lambda_2=2+\sqrt{7}, \lambda_3=2-\sqrt{7}$,

which by theory yeild nontrivial solutions. In particular for A^{-1}, the values of λ yieldings nontrivial solutions are

$$1, \frac{1}{2+\sqrt{7}}, \frac{1}{2-\sqrt{7}} \text{ } i.e., \text{ } 1, \frac{2+\sqrt{7}}{-3}, \frac{2-\sqrt{7}}{-3}$$

Hence (c) is false.

(27-29) :

27. (b) **28. (a)** **29. (c)**

Since the net movement must be two steps right (R) and two steps up (U). (2, 2) can be reached in 4 steps if the sequence of steps is some permutations of R, R, U, U.

$\therefore$ Probability of reaching (2, 2) in exactly 4 steps

$$=\frac{\left(\frac{4!}{2!2!}\right)}{4^4}=\frac{6}{4^4}$$

A six step sequence modules the steps R, R, U, U in same order as well as a pair of steps consisting of R, L or U, D in same order. R, R, U, U, R, L or R, R, U, U, U, D can be permuted in $2(60)$ ways of which 2×12 correspond to exactly 4 steps.

$\therefore$ Probability of reaching (2, 2) in exactly six steps

$$= \frac{2(60-12)}{4^6}=\frac{96}{4^6}$$

Probability of reaching (2, 2) in six or fewer steps

$$= \frac{6}{4^4}+\frac{96}{4^6}=\frac{3}{64}$$

30. (A)$\rightarrow$(r); (B)$\rightarrow$(s); (C)$\rightarrow$(p); (D)$\rightarrow$(q)

(A) $\left.\begin{matrix}2\log x-3\log y=1\\ 2\log x+3\log y=7\end{matrix}\right\}\Rightarrow \log x=2, \log y=1$

$\therefore$ $\log x + \log y = 3 \Rightarrow \log xy = 3$

(B) $$f(x)=\begin{cases}2, & -5\le x\le -2\\ |x|, & 1<|x|<2\\ x^2, & -1\le x\le 1\end{cases}$$

$\therefore$ Function is not derivable at $x = -2, -1, 1, 2$ *i.e.*, 4 points.

(C) Let $I=\int_0^2\frac{2x^3-6x^2+9x-5}{x^2-2x+5}dx$

Put $x - 1 = t$. Now when $x \to 0$, $t = -1$ and when $x \to 2$, $t \to 1$.

$$\therefore \quad I=\int_{-1}^{1}\frac{2(1+t)^3-6(1+t)^2+9(1+t)-5}{t^2+4}dt$$

$$=\int_{-1}^{1}\frac{2t^3+3t}{t^2+4}dt \Rightarrow I=0$$

(D) Using A.M. $\ge$ G.M. inequality, we have

$$\frac{4^{x^2}+4^{(x-1)^2}}{2}\ge\left(4^{x^2}\cdot 4^{(x-1)^2}\right)^{\frac{1}{2}}=2^{x^2}\cdot 2^{(x-1)^2}$$

$\Rightarrow$ $4^{x^2} + 4^{(x-1)^2} \ge 2^{x^2+(x-1)^2+1}$

Now $z = x^2 + (x - 1)^2 + 1$

$\Rightarrow$ $\frac{dz}{dx}=2x+2(x-1)=0$ for maximum or minimum

$\Rightarrow x=1/2$ $\therefore$ Min. value = 3/2

$\therefore$ $4^{x^2}+4^{(x-1)^2}$ has min. value $=2^{3/2}$

Hence $f(x)\ge\log_2(2^{3/2})=3/2$

$\because$ $y\ge 3/2 \Rightarrow$ Range is $\left[\frac{3}{2},\infty\right)$ $\Rightarrow$ $a=3/2$

MOCK TEST PAPER 2020

JEE MAIN

Exam Date : 05th, 07th to 09th & 11th April, 2020

With Numerical Value Type Questions

Time : 1 hr. 15 min. **Series - 8**

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

Unit No.	Topic	Syllabus In Details
UNIT NO.8	Integral calculus	Integral as limit of a sum. Fundamental theorem of calculus. Properties of definite integrals. Evaluation of definite integrals. Determining areas of the regions bounded by simple curves in standard form.
	Differential equation	Ordinary differential equations, their order and degree. Formation of differential equations. Solution of differential equations by the method of separation of variables. Solution of homogeneous and linear differential equation.
	Probability	Bayes' theorem. Probability distribution of a random variate. Bernoullian trials and Binomial distribution.

1. If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then the value of $P(X \geq 1)$ is

(a) $\frac{16}{81}$ (b) $\frac{65}{81}$ (c) $\frac{32}{81}$ (d) $\frac{37}{81}$

2. Let the population of rabbits surviving at a time be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t) =$

(a) $400 - 300e^{\frac{t}{2}}$ (b) $300 - 200e^{-\frac{t}{2}}$

(c) $600 - 500e^{\frac{t}{2}}$ (d) $400 - 300e^{-\frac{t}{2}}$

3. If $f(x) = \begin{cases} 3[x] - 5\frac{|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, then $\int_{-3/2}^{2} f(x)dx$ is equal to ([·] denotes the greatest integer function)

(a) $-\frac{11}{2}$ (b) $-\frac{7}{2}$ (c) -6 (d) $-\frac{17}{2}$

4. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, then $I_8 + I_6 =$

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

5. In a bolt factory three machines A, B and C manufacture respectively 2000, 2500 and 4000 bolts everyday. Of their outputs 3%, 4% and 2.5% are defective bolts. One bolt is drawn at random from a day's production and is found to be defective. Then the probability that it was produced by machine C is

(a) $\frac{3}{13}$ (b) $\frac{4}{13}$ (c) $\frac{5}{13}$ (d) $\frac{7}{13}$

6. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2\sin x] \, dx$, is

(a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

By : Sankar Ghosh, S.G.M.C, Mob. 09831244397.

7. The probability distribution of a random variable is

$$P(X)=\begin{cases}\frac{X}{15}; X=1,2,3,4,5\\ 0;\ \text{otherwise}\end{cases}.$$

Then $P\left\{\frac{1}{2}<X<\frac{5}{2}\mid X>1\right\}=$

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{2}{15}$ (d) $\frac{1}{15}$

8. The solution of differential equation $\frac{dy}{dx}+\frac{y}{x\log_e x}=\frac{1}{x}$ under the condition $y = 1$ when $x = e$ is

(a) $2y=\log_e x+\frac{1}{\log_e x}$ (b) $y=\log_e x+\frac{2}{\log_e x}$

(c) $y\log_e x=\log_e x+1$ (d) $y=\log_e x+e$

9. If $(x^2+y^2)dy=xydx$, $y(1)=1$ and $y(x_0)=e$, then $x_0=$

(a) $\sqrt{2(e^2-1)}$ (b) $\sqrt{2(e^2+1)}$

(c) $\sqrt{3}e$ (d) $\sqrt{\frac{e^2+1}{2}}$

10. The area of the region bounded by the parabola $(y-2)^2=(x-1)$, the tangent to the parabola at the point (2, 3) and the x-axis is

(a) 3 (b) 6 (c) 9 (d) 12

11. The graphs of $f(x)=x^2$ and $g(x)=cx^3$ intersect at two points. If the area of the region bounded between $f(x)$ and $g(x)$ over the interval $\left(0,\frac{1}{c}\right)$ equal to $\frac{2}{3}$, then the value of $\frac{1}{c}+\frac{1}{c^2}$ is

(a) 20 (b) 2 (c) 6 (d) 12

12. If X is a binomial variate with parameter n and p where $0<p<1$ such that $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then p equals

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{7}$

13. The mean and variance of a binomial distribution are 4 and 3 respectively. Then probability of getting exactly six success in this distribution, is

(a) ${}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$ (b) ${}^{16}C_6\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^{20}$

(c) ${}^{16}C_6\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^{12}$ (d) ${}^{16}C_9\left(\frac{1}{4}\right)^{16}\left(\frac{3}{4}\right)^{20}$

14. For a fixed value of n, the maximum value of variance of binomial distribution is

(a) $\frac{n}{4}$ (b) $\frac{n}{2}$

(c) $\frac{n}{8}$ (d) None of these

15. A random variable X has the following distribution:

X	3	4	5	6
$P(X)$	0.2	0.4	0.3	0.1

Then the value of $E(X^2)$ is

(a) 18.6 (b) 19.6 (c) 18.3 (d) 19.3

16. The solution of the differential equation

$$y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$$ is (where c is a constant)

(a) $\phi\left(\frac{y^2}{x^2}\right)=cx$ (b) $x\phi\left(\frac{y^2}{x^2}\right)=c$

(c) $\phi\left(\frac{y^2}{x^2}\right)=cx^2$ (d) $x^2\phi\left(\frac{y^2}{x^2}\right)=c$

17. Solution of the differential equation

$\tan y\frac{dy}{dx}=\sin(x+y)+\sin(x-y)$ is

(a) $\sec y-2\cos x=c$ (b) $\sec y+2\cos x=c$

(c) $\cos y-2\sin x=c$ (d) $\sec y+2\sin x=c$

18. If $A=\int_0^{\pi}\frac{\sin x}{\sin x+\cos x}dx$, $B=\int_0^{\pi}\frac{\sin x}{\sin x-\cos x}dx$ then

(a) $A+B=0$ (b) $A-B=\pi$

(c) $A=B=\frac{\pi}{2}$ (d) $A=-B=\pi$

19. The value of the integral $\int_0^{\pi/2}\log\left|\tan x+\cot x\right|dx$ is

(a) $\pi\log 2$ (b) $-\pi\log 2$

(c) $\frac{\pi}{2}\log 2$ (d) None of these

20. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is

(a) 0.534 (b) 0.466 (c) 0.763 (d) 0.546

NUMERICAL VALUE TYPE

21. Let $f(x) = ax^3 + bx^2 + cx$ has relative extrema at $x = 1$ and at $x = 5$. If $\int_{-1}^{1} f(x)\,dx = 6$, then $-(a+b+c) =$

22. The solution of the differential equation $e^{\frac{dy}{dx}} = x+1$, when $y(0) = 3$, is $y = (x+1)\log|x+1| - x + k$. What is the value of k?

23. A curve passes through (2, 0) and slope of the tangent at the point $(x, y) = \dfrac{(x+1)^2 + y - 3}{x+1}$. The area bounded by the curve and the x-axis is $\dfrac{k}{3}$. What is the value of k?

24. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)\,dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

25. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is

SOLUTIONS

1. (b) : Here, we have $np = \dfrac{4}{3}$ and $npq = \dfrac{8}{9}$

$\therefore \quad \dfrac{npq}{np} = \dfrac{8}{9} \times \dfrac{3}{4} = \dfrac{2}{3} \Rightarrow q = \dfrac{2}{3} \quad \therefore \; p = \dfrac{1}{3}$ and $n = 4$

$$P(X = x) = \binom{n}{x} p^x q^{n-x} = \binom{4}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{4-x}$$

Now, $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$$= 1 - \binom{4}{0}\left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{4-0} = 1 - \frac{16}{81} = \frac{65}{81}$$

2. (a) : Given, $\dfrac{dp(t)}{dt} = \dfrac{1}{2}p(t) - 200$

$$\Rightarrow \quad \frac{dp(t)}{dt} = \frac{p(t) - 400}{2}$$

Intergrating both sides, we have $\displaystyle\int \frac{dp(t)}{p(t) - 400} = \frac{1}{2}\int dt$

$$\Rightarrow \quad \log|p(t) - 400| = \frac{1}{2}t + \log c$$

$$\Rightarrow \quad \log\left|\frac{p(t) - 400}{c}\right| = \frac{1}{2}t \quad \Rightarrow \quad p(t) = ce^{\frac{t}{2}} + 400$$

When $p(0) = 100$, we get $100 = ce^0 + 400 \Rightarrow c = -300$

$\therefore$ The required solution is $p(t) = 400 - 300e^{\frac{t}{2}}$.

3. (a) : Here, $3[x] - 5\dfrac{|x|}{x} = \begin{cases} 3[x] - 5, & \text{if } x > 0 \\ 3[x] + 5, & \text{if } x < 0 \end{cases}$

$$\therefore \int_{-3/2}^{2} f(x)dx = \int_{-3/2}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx$$

$$= \int_{-3/2}^{-1} (3\times(-2)+5)dx + \int_{-1}^{0} (-3+5)dx + \int_{0}^{1} (0-5)dx + \int_{1}^{2} (-2)dx$$

$$= [-x]_{-3/2}^{-1} + 2[x]_{-1}^{0} - 5[x]_{0}^{1} - 2[x]_{1}^{2}$$

$$= -1\left(-1 + \frac{3}{2}\right) + 2(1) + 1(-5) + (-2)1 = -\frac{11}{2}$$

4. (d) : Given, $I_n = \displaystyle\int_0^{\pi/4} \tan^n x\,dx$

$$\therefore \quad I_8 + I_6 = \int_0^{\pi/4} (\tan^8 x + \tan^6 x)\,dx$$

$$= \int_0^{\pi/4} \tan^6 x(\tan^2 x + 1)\,dx = \int_0^{\pi/4} \tan^6 x\,d(\tan x)$$

$$\Rightarrow \left[\frac{\tan^7 x}{7}\right]_0^{\pi/4} = \frac{1}{7}$$

5. (c) : Let us define the events in the following way:

E_1 : The bolts are manufactured in machine A

E_2 : The bolts are manufactured in machine B

E_3 : The bolts are manufactured in machine C

F : The selected bolt is defective.

$$\therefore \quad P(E_1) = \frac{2000}{8500} = \frac{4}{17}, \; P(E_2) = \frac{2500}{8500} = \frac{5}{17}$$

PUZZLE CORNER

ANSWER · FEBRUARY 2020

4− 2	24× 6	1	4	8+ 3	5
6	12× 2	3	20× 1	5	4
1− 4	3	2	11+ 5	6	3÷ 1
1− 5	4	120× 6	2÷ 2	1	3
3÷ 1	5	4	36× 3	2	6
3	1	11+ 5	6	2− 4	2

and $P(E_3)=\frac{4000}{8500}=\frac{8}{17}$

$P(F|E_1)=\frac{3}{100}$, $P(F|E_2)=\frac{4}{100}$ and $P(F|E_3)=\frac{2.5}{100}$

Now, $P(E_3|F)=\frac{P(E_3)P(F|E_3)}{P(E_1)P(F|E_1)+P(E_2)P(F|E_2)+P(E_3)P(F|E_3)}$

$=\frac{\frac{8}{17}\times\frac{2.5}{100}}{\frac{4}{17}\cdot\frac{3}{100}+\frac{5}{17}\cdot\frac{4}{100}+\frac{8}{17}\cdot\frac{2.5}{100}}=\frac{20}{52}=\frac{5}{13}$

6. (c) : Let $I=\int_{\pi/2}^{3\pi/2}[2\sin x]\,dx$

$=\int_{\pi/2}^{5\pi/6}[2\sin x]\,dx+\int_{5\pi/6}^{\pi}[2\sin x]\,dx+\int_{\pi}^{7\pi/6}[2\sin x]\,dx+\int_{7\pi/6}^{3\pi/2}[2\sin x]\,dx$

$=\int_{\pi/2}^{5\pi/6}1\,dx+\int_{5\pi/6}^{\pi}0\,dx+\int_{\pi}^{7\pi/6}(-1)\,dx+\int_{7\pi/6}^{3\pi/2}(-2)\,dx$

$=\left(\frac{5\pi}{6}-\frac{\pi}{2}\right)-\left(\frac{7\pi}{6}-\pi\right)-2\left(\frac{3\pi}{2}-\frac{7\pi}{6}\right)=-\frac{\pi}{2}$

7. (b) : $P(X)=\begin{cases}\frac{X}{15}; X=1,2,3,4,5\\ 0, \text{ otherwise}\end{cases}$

Now, $P\left(\frac{1}{2}<X<\frac{5}{2}\,|\,X>1\right)=\frac{P\left\{\left(\frac{1}{2}<X<\frac{5}{2}\right)\cap X>1\right\}}{P(X>1)}$

$=\frac{P\{(X=1 \text{ or } 2)\cap(X>1)\}}{P(X>1)}=\frac{P(X=2)}{1-P(X=1)}=\frac{\frac{2}{15}}{1-\frac{1}{15}}=\frac{1}{7}$

8. (a) : The given equation is $\frac{dy}{dx}+\frac{y}{x\log_e x}=\frac{1}{x}$

Here, I.F. $=e^{\int\frac{dx}{x\log_e x}}=e^{\log(\log_e x)}=\log_e x$

$\therefore$ Solution is $y\log_e x=\int\frac{1}{x}\log_e x\,dx$

$\Rightarrow\quad y\log_e x=\int\log_e x\,d(\log_e x)$

$\Rightarrow\quad y\log_e x=\frac{(\log_e x)^2}{2}+c$ ...(i)

Given, $y=1$ when $x=e$

$\therefore$ From (i), we get $1\times1=\frac{1}{2}+c\Rightarrow c=\frac{1}{2}$

Thus (i) becomes

$$y\log_e x=\frac{(\log_e x)^2}{2}+\frac{1}{2}\Rightarrow 2y=\log_e x+\frac{1}{\log_e x}$$

9. (c) : We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Putting $y=vx$, we get

$v+x\frac{dv}{dx}=\frac{v}{1+v^2}\Rightarrow x\frac{dv}{dx}=\frac{v}{1+v^2}-v=-\frac{v^3}{1+v^2}$

$\Rightarrow\quad\frac{1+v^2}{v^3}dv=-\frac{dx}{x}\Rightarrow\int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$

$\Rightarrow\quad\ln x+\ln v-\frac{1}{2v^2}=c\Rightarrow\ln y=\frac{x^2}{2y^2}+c$

Now, $x=1, y=1\Rightarrow c=-\frac{1}{2}$

Again $y=e$, $x=x_0\Rightarrow 1=\frac{x_0^2}{2e^2}-\frac{1}{2}$

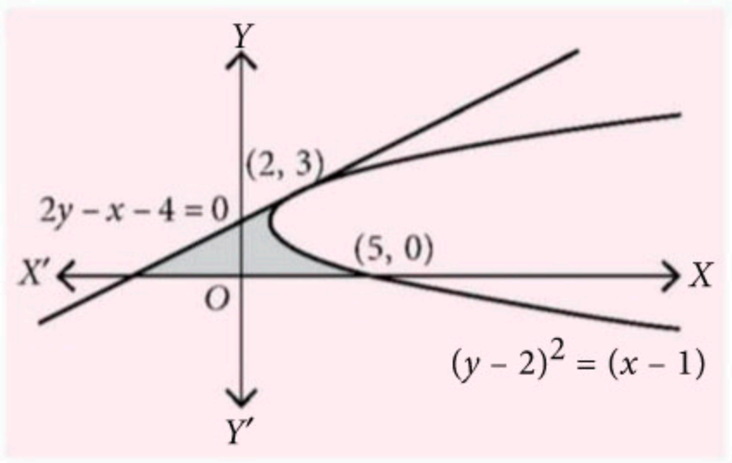
$\Rightarrow\quad x_0^2=3e^2\Rightarrow x_0=\sqrt{3}e$

10. (c) : The given equation of the parabola is

$$y^2-4y-x+5=0$$

The equation of the tangent at (2, 3) is

$$3y-2(y+3)-\frac{x+2}{2}+5=0 \text{ or } 2y-x-4=0$$

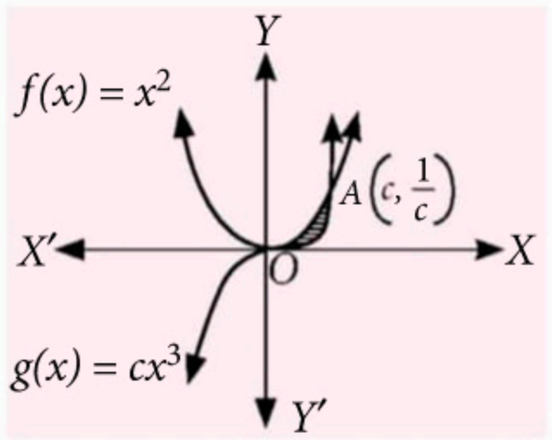


Therefore the required area is

$$\int_0^3(x_2-x_1)\,dx=\int_0^3[\{(y-2)^2+1\}-\{2y-4\}]\,dy$$

$$=\int_0^3(y^2-6y+9)dy=\int_0^3(3-y)^2\,dy=-\left[\frac{(3-y)^3}{3}\right]_0^3=9$$

11. (c) : The two curves $f(x)=x^2$ and $g(x)=cx^3$ intersect at $A\left(c,\frac{1}{c}\right)$ and $O(0,0)$.



$$\therefore \quad \int_0^{1/c} (y_1 - y_2)\,dx = \frac{2}{3} \Rightarrow \int_0^{1/c} (x^2 - cx^3)\,dx = \frac{2}{3}$$

$$\Rightarrow \left[\frac{x^3}{3} - \frac{cx^4}{4}\right]_0^{1/c} = \frac{2}{3} \Rightarrow \frac{1}{12c^3} = \frac{2}{3} \Rightarrow c = \frac{1}{2}$$

Hence, $\frac{1}{c} + \frac{1}{c^2} = 2 + 4 = 6$

12. (a) : Let a random variable X follows a binomial distribution with parameter n and p.

Consider $\frac{P(X=r)}{P(X=n-r)} = \frac{{}^nC_r p^r (1-p)^{n-r}}{{}^nC_{n-r} p^{n-r} (1-p)^r}$

$$\Rightarrow \quad \frac{P(X=r)}{P(X=n-r)} = \left(\frac{1-p}{p}\right)^{n-2r} = \left(\frac{1}{p} - 1\right)^{n-2r}$$

For this to be independent of n and r, we must have

$$\frac{1}{p} - 1 = 1 \Rightarrow p = \frac{1}{2}$$

13. (a) : Given that $X \sim B(n, p)$ with $np = 4$

and $npq = 3 \Rightarrow q = \frac{3}{4}, p = \frac{1}{4}$ and $n = 16$

$$\therefore \quad P(X=r) = {}^{16}C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{16-r}, \quad r = 0,1,...,16$$

$$\Rightarrow \quad P(X=6) = {}^{16}C_6 \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^{10}$$

14. (a) : We know, that the variance of binomial distribution is, $\text{Var}(x) = npq = n\left[\left(\frac{p+q}{2}\right)^2 - \left(\frac{p-q}{2}\right)^2\right]$

$$= n\left[\frac{1}{4} - \left(\frac{p-q}{2}\right)^2\right] \le \frac{n}{4} \qquad \left[\because \quad \left(\frac{p-q}{2}\right)^2 \ge 0\right]$$

Hence, for a fixed value of n, the variance of the binomial distribution can never exceed $n/4$. So, the maximum value of var(X) is $n/4$.

15. (d) : $E(X^2) = 3^2 \times 0.2 + 4^2 \times 0.4 + 5^2 \times 0.3 + 6^2 \times 0.1 = 19.3$

16. (c) : $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$

Putting $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \quad vx\left(v + x\frac{dv}{dx}\right) = x\left[v^2 + \frac{\phi(v^2)}{\phi'(v^2)}\right]$$

$$\Rightarrow \quad v^2 + vx\frac{dv}{dx} = v^2 + \frac{\phi(v^2)}{\phi'(v^2)} \Rightarrow v\frac{\phi'(v^2)}{\phi(v^2)}dv = \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{dx}{x} = \int \frac{v\phi'(v^2)}{\phi(v^2)}dv$$

$$\Rightarrow \quad \ln x + \ln k = \frac{1}{2}\ln\{\phi(v^2)\} \Rightarrow \ln kx = \ln\{\phi(v^2)\}^{1/2}$$

$$\Rightarrow \quad kx = \{\phi(v^2)\}^{1/2} \Rightarrow k^2x^2 = \phi(v^2) \Rightarrow cx^2 = \phi\left(\frac{y^2}{x^2}\right)$$

17. (b) : The given differential equation is

$$\tan y\frac{dy}{dx} = \sin(x+y) + \sin(x-y) = 2\sin x\cos y$$

$$\Rightarrow \quad \tan y\,\sec y dy = 2\sin x dx$$

$$\Rightarrow \quad \int \tan y \sec y dy = 2\int \sin x dx$$

$$\Rightarrow \quad \sec y = -2\cos x + c \Rightarrow \sec y + 2\cos x = c$$

18. (c) : Given, $A = \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}\,dx$

$$= \int_0^{\pi} \frac{\sin(\pi - x)}{\sin(\pi - x) + \cos(\pi - x)}\,dx = \int_0^{\pi} \frac{\sin x}{\sin x - \cos x}\,dx = B$$

Now, $A + B = \int_0^{\pi} \frac{2\sin^2 x}{\sin^2 x - \cos^2 x}\,dx = \int_0^{\pi} \frac{1 - \cos 2x}{-\cos 2x}\,dx$

$$= \int_0^{\pi} (1 - \sec 2x)\,dx = \pi - \left[\frac{\log(\sec 2x + \tan 2x)}{2}\right]_0^{\pi} = \pi$$

Hence, $A = B = \frac{\pi}{2}$

19. (a) : Let $I = \int_0^{\pi/2} \log\left|\tan x + \cot x\right| dx$

$$= \int_0^{\pi/2} \log\left|\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right| dx$$

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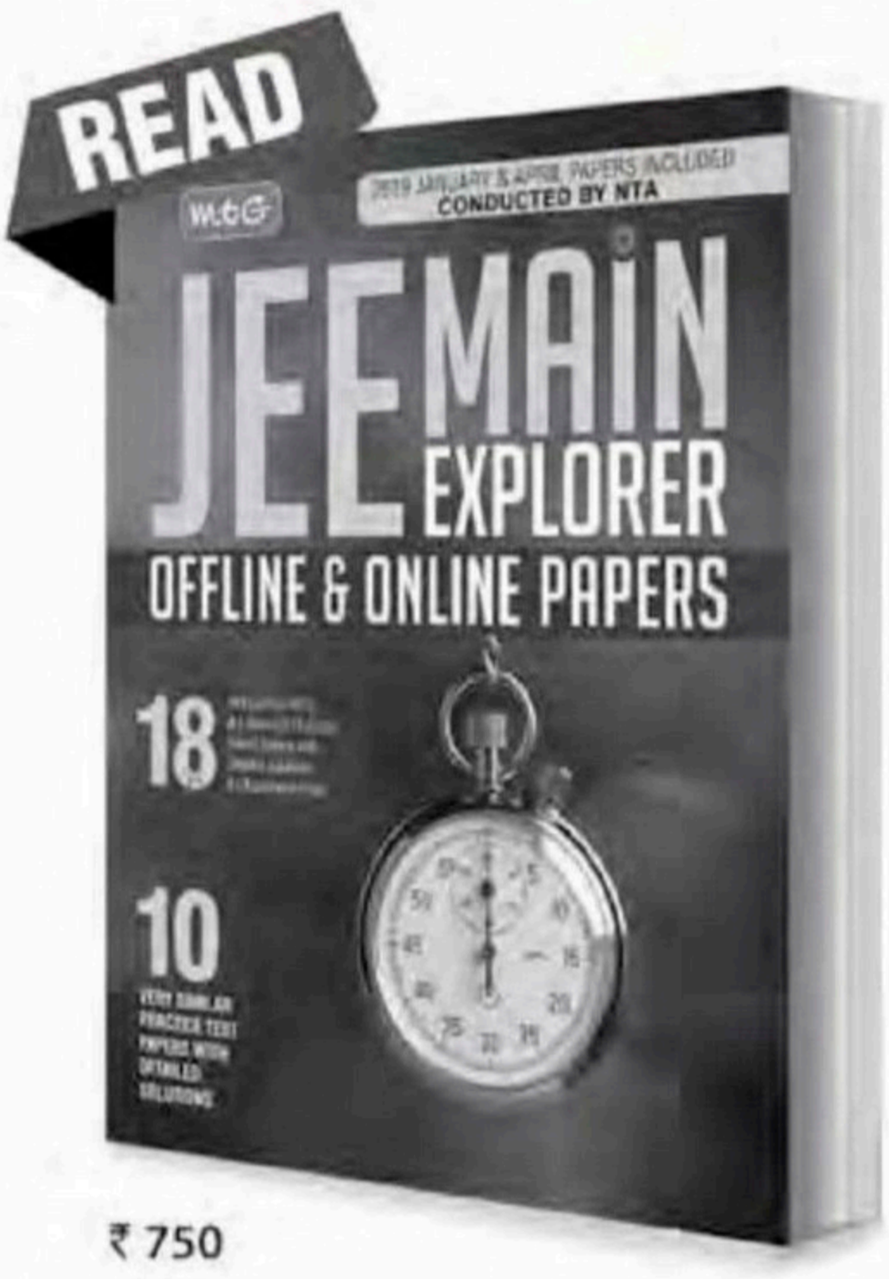

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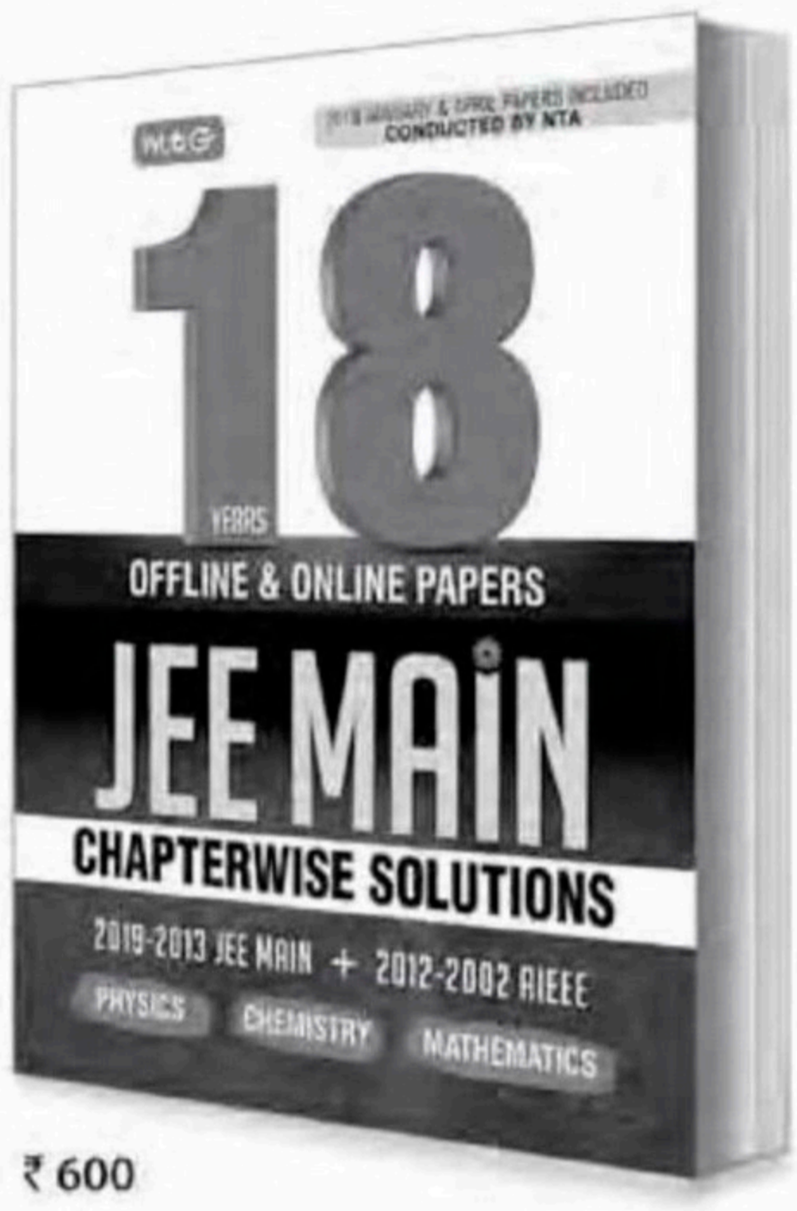

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$$= -\int_0^{\pi/2} \log \sin x \, dx - \int_0^{\pi/2} \log \cos x \, dx = \pi \log 2$$

20. (a) : Let p denotes the probability of a correct distinction between a cup of instant coffee and a cup of percolator coffee.

$$\therefore \quad p = \frac{75}{100} = \frac{3}{4} \Rightarrow q = 1 - p = \frac{1}{4} \text{ and } n = 6$$

If the random variable X denotes the number of correct distinctions, then by the binomial probability law, the probability of x correct identifications out of 6 cups is given by :

$$P(X = x) = {}^6C_x \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{6-x} ; x = 0, 1, ..., 6$$

The probability of the claim being accepted is

$P(X \geq 5) = P(5) + P(6)$

$$= {}^6C_5 \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^{6-5} + {}^6C_6 \left(\frac{3}{4}\right)^6 = 0.534$$

21. (7) : Here, $f(x) = ax^3 + bx^2 + cx$

$\therefore \quad f'(x) = 3ax^2 + 2bx + c$

Now, we have $f'(1) = 0$ and $f'(5) = 0$

[Since $f(x)$ has extrema at $x = 1$ and at $x = 5$]

$\Rightarrow \quad 3a + 2b + c = 0$ ...(i) and $75a + 10b + c = 0$ ...(ii)

Again $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (ax^3 + bx^2 + cx)\,dx$

$$= \left[a\frac{x^4}{4} + b\frac{x^3}{3} + c\frac{x^2}{2} \right]_{-1}^{1} = b\left[\frac{1}{3} - \left(\frac{-1}{3}\right)\right] = \frac{2b}{3}$$

$$\therefore \quad \frac{2b}{3} = 6 \quad \Rightarrow \quad b = 9$$

From (i) and (ii), we get $3a + 18 + c = 0$

and $75a + 90 + c = 0$

Solving these equations, we get $a = -1$, $c = -15$

$\therefore \quad -(a + b + c) = -(-1 + 9 - 15) = 7$

22. (3) : The given differential equation is $e^{\frac{dy}{dx}} = x + 1$

$$\Rightarrow \quad \frac{dy}{dx} = \log_e (x+1) \quad \Rightarrow \quad \int dy = \int \log_e (x+1)\,dx$$

$\Rightarrow \quad y = (x + 1)\log|x + 1| - (x + 1) + c$

When $y(0) = 3$, then $3 = 0 - 1 + c \Rightarrow c = 4$

$\Rightarrow \quad y = (x + 1)\log|x + 1| - x + 3$

23. (4) : The given differential equation is

$$\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = (x+1) + \frac{y-3}{x+1}$$

Let $x + 1 = X$ and $y - 3 = Y$

$$\therefore \quad \frac{dY}{dX} - \frac{Y}{X} = X \quad \Rightarrow \quad \text{I.F.} = \frac{1}{X}$$

General solution is $\frac{Y}{X} = X + c$

$$\Rightarrow \quad \frac{y-3}{x+1} = x + 1 + c \qquad ...(i)$$

When $x = 2$, $y = 0$, then $c = -4$

Now (i) becomes $y - 3 = (x + 1)^2 - 4(x + 1) = x^2 - 2x - 3$

$\Rightarrow \quad y = x^2 - 2x$ meets x-axis at $x = 0, 2$

$$\text{Area} = \int_0^2 (2x - x^2)dx = 4 - \frac{8}{3} = \frac{4}{3}$$

24. (6) : We have $6\int_1^x f(t)\,dt = 3xf(x) - x^3$

Differentiating both sides with respect to x, we get

$6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$$\Rightarrow \quad 3f(x) = 3xf'(x) - 3x^2 \Rightarrow \quad f'(x) - \frac{1}{x} f(x) = x$$

$$\Rightarrow \quad \frac{d}{dx}\{f(x)\} + \left(-\frac{1}{x}\right) f(x) = x$$

$$\text{Here I.F.} = e^{\int -\frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

$$\therefore \quad \text{General solution is } \frac{f(x)}{x} = x + c$$

$\Rightarrow \quad f(x) = x^2 + cx$

$\Rightarrow \quad f(1) = 1 + c \Rightarrow 2 = 1 + c \Rightarrow c = 1$

Thus, $f(x) = x^2 + x$ $\therefore$ $f(2) = 4 + 2 = 6$

25. (8) : Let the coin be tossed n times and X be the random variable denotes number of heads obtained in n tosses of the coin. Then

$$P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r} = {}^nC_r \left(\frac{1}{2}\right)^n ; r = 0, 1, 2..., n$$

Given that, $P(X \geq 2) \geq 0.96 \Rightarrow \{1 - P(X < 2)\} \geq 0.96$

$\Rightarrow \quad \{1 - P(X = 0) - P(X = 1)\} \geq 0.96$

$$\Rightarrow \quad \left(1 - {}^nC_0 \left(\frac{1}{2}\right)^n - {}^nC_1 \left(\frac{1}{2}\right)^n\right) \geq 0.96$$

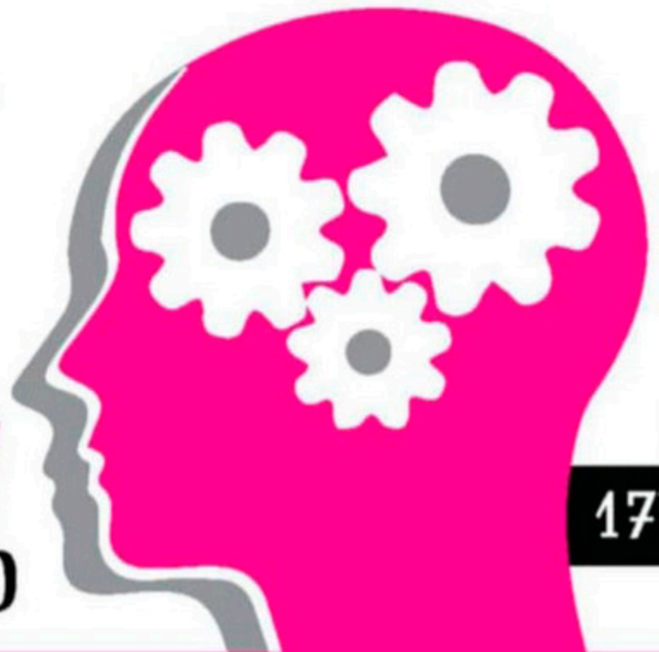
$$\Rightarrow \quad \frac{1}{2^n}(2^n - {}^nC_0 - {}^nC_1) \geq 0.96 \Rightarrow 1 - \frac{n+1}{2^n} \geq 0.96$$

$\Rightarrow \quad (n + 1) \leq (0.04)2^n \Rightarrow n = 8, 9, 10$,.....

Hence the least value of n is 8.

PRACTICE PROBLEMS

JEE ADVANCED 2020



Exam Date
17th May, 2020

SECTION-I

One or More Than One Option(s) Correct Type

1. The maximum value of $(\cos\alpha_1)\cdot(\cos\alpha_2)\ldots\ldots(\cos\alpha_n)$ under the restrictions $0 \le \alpha_1, \alpha_2, \ldots\ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1)\cdot(\cot\alpha_2)\ldots\ldots\ldots\ldots(\cot\alpha_n) = 1$ is

(a) $\frac{1}{2^{n/2}}$ (b) $\frac{1}{2^n}$ (c) 2^n (d) 1

2. Minimum distance between the vertices of $y = x^2 - 2ax + 1$ and $y = x^2 - ax + \frac{3a^2}{4}$ occurs when 'a' is

(a) $\frac{\sqrt{11}}{3\sqrt{2}}$ (b) $-\sqrt{5}$ (c) $\sqrt{5}$ (d) $-\frac{\sqrt{11}}{3\sqrt{2}}$

3. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(ω) denotes the principal argument of a non-zero complex number ω, then

(a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(b) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(c) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(d) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

4. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(a) $\binom{n+1}{r-1}$ (b) $2\binom{n+1}{r+1}$

(c) $\binom{n+2}{r}$ (d) $2\binom{n+2}{r}$

In ΔABC, internal angle bisector of $\angle A$ meets side BC in D. $DE \perp AD$ meets AC in E and AB in F, then

(a) AE is H.M. of b and c

(b) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$ (c) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$

(d) ΔAEF is an isosceles triangle.

6. If $I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ and

$I_2 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2}$, then $\frac{I_1}{I_2}$ is

(a) 1/2 (b) 2

(c) 0 (d) none of these

7. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4}\ldots\ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\ldots\right) = \frac{\pi}{2}$

for $0 < |x| < \sqrt{2}$, then x equals

(a) 1/2 (b) 1 (c) −1/2 (d) − 1

8. The solution of $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{a^2 - x^2 - y^2}{x^2 + y^2}}$ is

(a) $\sqrt{x^2 + y^2} = a\sin\left[\left(\tan^{-1}\frac{y}{x}\right) + c\right]$

(b) $\sqrt{x^2 + y^2} = a\left(\cos\left(\tan^{-1}\frac{y}{x}\right) + c\right)$

(c) $\sqrt{x^2 + y^2} = a\left(\tan\left(\sin^{-1}\frac{y}{x}\right) + c\right)$

(d) $y = x\tan\left(c + \sin^{-1}\frac{1}{a}\sqrt{x^2 + y^2}\right)$

9. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and $\vec{r}$ be any vector in the space, then $[\vec{b}\ \vec{c}\ \vec{r}]\,\vec{a} + [\vec{c}\ \vec{a}\ \vec{r}]\vec{b} + [\vec{a}\ \vec{b}\ \vec{r}]\vec{c} =$

(a) $[\vec{c}\ \vec{a}\ \vec{b}]\,\vec{r}$ (b) $[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$

(c) $2[\vec{a}\ \vec{b}\ \vec{c}]\,\vec{r}$ (d) $[\vec{b}\ \vec{c}\ \vec{a}]\,\vec{r}$

10. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true?

(a) $p + m + c = 19/20$ (b) $p + m + c = 27/20$

(c) $pmc = 1/10$ (d) $pmc = 1/4$

SECTION-II

Comprehension Type

Paragraph for Q. No. 11 to 13

If $f : R \to [0, \infty)$ be a function satisfying the property $f(x + y) - f(x - y) = f(x)[f(y) - f(-y)]$ for all real x and y, $f'(0) = \log a$, $f(0) = 1$.

11. $f(x)$ is

(a) e^x (b) $2\ln x$ (c) $4x$ (d) a^x

12. $f'(x)$ is

(a) e^x (b) $a^x \log a$ (c) 4 (d) $5x$

13. The solution of differential equation

$$\frac{dy}{dx} = \frac{(\log_a f(x) + \log_a f(y))^2}{[\log_a f(x) + 2][\log_a f(y) - 2]}$$ is

(a) $\dfrac{y-2}{x+2} - \dfrac{1}{2}\ln\left|1 + 2\left(\dfrac{y-2}{x+2}\right)\right| = 2\log(x+2) + c$

(b) $\dfrac{y+2}{x-2} - \ln\left|1 + \dfrac{y+2}{x-2}\right| = c$

(c) $(x+2)(y-2) = \ln\left(\dfrac{x+2}{y-2}\right) + c$

(d) None of these

Matrix-Match Type

14. Match the following :

	List I	List II	
P.	Number of ordered pairs (x, y) satisfying $x^2 + 1 = y$ and $y^2 + 1 = x$ simultaneoulsy is	1.	1
Q.	If $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$ $\forall$ $A, B < \dfrac{\pi}{2}$, then $\sin(A + 2B)$ equals	2.	55
R.	The distance of the point $(1, 0, -3)$ from the plane $x - y - z = 9$ measured parallel to the line $\dfrac{x-2}{2} = \dfrac{y+2}{3} = \dfrac{z-6}{-6}$ is	3.	7
		4.	0

SECTION-III

Numerical Value Type

15. Let $a_1, a_2, a_3, \ldots\ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\dfrac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\dfrac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to

16. If $f(x) = \cos\left(\dfrac{\pi}{2}[x] - x^3\right)$, $1 < x < 2$, where $[\cdot]$ is the greatest integer function, then $f'\left(\sqrt[3]{\dfrac{\pi}{3}}\right)$ is

17. If $f(x) = \displaystyle\int \frac{3(\ln x)^2 + (\ln x)^4}{x(1 + (\ln x)^2 - (\ln x)^3)^2}\,dx$ and $f(1) = 0$, then the value of $|3f(e^2)|$ is equal to

18. If $f(x) = \begin{cases} x - [x+2] + \dfrac{5}{2} & \text{if } x \notin I \\ \dfrac{1}{2} & \text{if } x \in I \end{cases}$, where $[\cdot]$ denotes greatest integer function and $g(x) = \max. [f(x), x^2, x]$, $0 \le x \le 2$ and area between $g(x)$ and $y = 0$ is A, then $[A]$ is

SOLUTIONS

1. (a) : Given, $\cot\alpha_1 \cot\alpha_2 \ldots\ldots\ldots \cot\alpha_n = 1$

$\Rightarrow$ $\cos\alpha_1 \cos\alpha_2 \ldots\ldots \cos a_n = \sin\alpha_1 \sin\alpha_2 \ldots\ldots\ldots \sin\alpha_n$

$\Rightarrow$ $(\cos\alpha_1 \cos\alpha_2 \ldots\ldots\ldots \cos\alpha_n)^2$

$= (\cos\alpha_1 \sin\alpha_1)(\cos\alpha_2 \sin\alpha_2) \ldots\ldots (\cos\alpha_n \sin\alpha_n)$

$= \dfrac{1}{2^n}(\sin 2\alpha_1)(\sin 2\alpha_2) \ldots\ldots (\sin 2\alpha_n) \le \dfrac{1}{2^n}$

: First vertex is $(a, 1 - a^2)$ and second vertex is $\left(\dfrac{a}{2}, \dfrac{a^2}{2}\right)$. So, distance $= \sqrt{\dfrac{a^2}{4} + \left(1 - \dfrac{3a^2}{2}\right)^2}$

Let $a^2 = t \Rightarrow$ distance $= \sqrt{\dfrac{t}{4} + 1 + \dfrac{9t^2}{4} - 3t} = \sqrt{f(t)}$ (say)

$\therefore$ $f'(t) = \dfrac{1}{4} - 3 + \dfrac{9}{2}t = 0 \Rightarrow \dfrac{9t}{2} = \dfrac{11}{4} \Rightarrow t = \dfrac{11}{18}$

$\Rightarrow a = \pm\dfrac{\sqrt{11}}{3\sqrt{2}}$

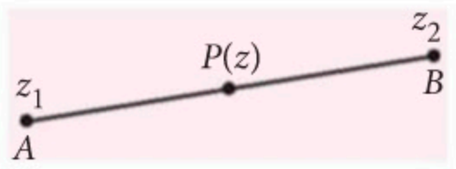
3. (a, c, d) : The given statement implies that z is on the line segment joining $A(z_1)$ and $B(z_2)$.

$\therefore$ $|z - z_1| + |z - z_2| = |z_1 - z_2|$

We have, $z = (1 - t)z_1 + tz_2$

Now z_1, z, z_2 are collinear,

so $\arg\left(\dfrac{z - z_1}{z_2 - z_1}\right) = 0$



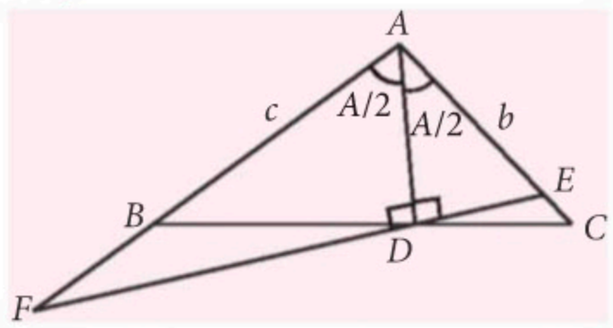
$\Rightarrow$ $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

Also, $\dfrac{z-z_1}{\bar{z}-\bar{z}_1}=\dfrac{z_2-z_1}{\bar{z}_2-\bar{z}_1}$ complex slope of the line which can be rearranged as $(z-z_1)(\bar{z}_2-\bar{z}_1)-(z_2-z_1)(\bar{z}-\bar{z}_1)=0$

i.e., $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix}=0$

4. (c)

5. (a, b, c, d) :



We have, $\text{ar}(\Delta ABC) = \text{ar}(\Delta ABD) + \text{ar}(\Delta ADC)$

$\Rightarrow \quad \dfrac{1}{2}bc\sin A=\dfrac{1}{2}c\,AD\sin\dfrac{A}{2}+\dfrac{1}{2}b\,AD\sin\dfrac{A}{2}$

$\Rightarrow \quad AD=\dfrac{2bc\cos\dfrac{A}{2}}{b+c}$ ...(i)

Also, $AD=AE\cos\dfrac{A}{2}\Rightarrow AE=\dfrac{2bc}{b+c}=$ H.M. of b and c

Again $EF = DE + DF = 2DE$

($\because$ $AD \perp EF$ and AD is bisector so $DE = DF$)

$=2AD\tan\dfrac{A}{2}=\dfrac{4\,bc\sin\dfrac{A}{2}}{b+c}$ [Using (i)]

By simple geometry in ΔAFE, $AF = AE$

$\therefore$ ΔAFE is an isosceles triangle.

6. (a) : $I_1=\int\limits_{-100}^{101}\dfrac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$

$\Rightarrow \quad I_1=\int\limits_{-100}^{101}\dfrac{dx}{(5+2(1-x)-2(1-x)^2)(1+e^{2-4(1-x)})}$

$\Rightarrow \quad 2I_1=\int\limits_{-100}^{101}\dfrac{dx}{5+2x-2x^2}=I_2\Rightarrow\dfrac{I_1}{I_2}=\dfrac{1}{2}$

7. (b) : $x-\dfrac{x^2}{2}+\dfrac{x^3}{4}-.....=x\left(\dfrac{1}{1+\dfrac{x}{2}}\right)=\dfrac{2x}{2+x}$

$x^2-\dfrac{x^4}{2}+\dfrac{x^6}{4}-.....=\dfrac{x^2}{1+\dfrac{x^2}{2}}=\dfrac{2x^2}{2+x^2}$

$\therefore \quad \sin^{-1}\left(\dfrac{2x}{2+x}\right)+\cos^{-1}\left(\dfrac{2x^2}{2+x^2}\right)=\dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{2x}{2+x}=\dfrac{2x^2}{2+x^2}\Rightarrow x=1$

8. (a, d) : Taking, $x = r\cos\theta$ and $y = r\sin\theta$ so that $x^2 + y^2 = r^2$ and $\dfrac{y}{x}=\tan\theta$, we have $x\,dx + y\,dy = rdr$ and $xdy - ydx = x^2\sec^2\theta\,d\theta = r^2\,d\theta$

The given equation can be transformed into

$\dfrac{rdr}{r^2d\theta}=\sqrt{\dfrac{a^2-r^2}{r^2}}\Rightarrow\dfrac{dr}{d\theta}=\sqrt{a^2-r^2}$

$\Rightarrow \quad c+\sin^{-1}\dfrac{r}{a}=\theta=\tan^{-1}\dfrac{y}{x}$

$\Rightarrow \quad y=x\tan\left(c+\sin^{-1}\dfrac{1}{a}\sqrt{x^2+y^2}\right)$

or $\quad \sqrt{x^2+y^2}=a\sin\left(c+\tan^{-1}\dfrac{y}{x}\right)$

9. (a, b, d) : $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{r})=\vec{A}\times(\vec{c}\times\vec{r})$, where $\vec{A}=\vec{a}\times\vec{b}$

Now, $\vec{A}\times(\vec{c}\times\vec{r})=(\vec{A}\cdot\vec{r})\vec{c}-(\vec{A}\cdot\vec{c})\vec{r}$

$=[\vec{a}\;\vec{b}\;\vec{r}]\,\vec{c}-[\vec{a}\;\vec{b}\;\vec{c}]\,\vec{r}$

Similarly, $(\vec{a}\times\vec{b})\times\vec{B}=[\vec{a}\;\vec{c}\;\vec{r}]\vec{b}-[\vec{b}\;\vec{c}\;\vec{r}]\vec{a}$, where $\vec{B}=\vec{c}\times\vec{r}$

So, $[\vec{a}\;\vec{b}\;\vec{r}]\,\vec{c}-[\vec{a}\;\vec{b}\;\vec{c}]\vec{r}=[\vec{a}\;\vec{c}\;\vec{r}]\vec{b}-[\vec{b}\;\vec{c}\;\vec{r}]\vec{a}$

$\Rightarrow \quad [\vec{a}\;\vec{b}\;\vec{r}]\,\vec{c}-[\vec{a}\;\vec{b}\;\vec{c}]\vec{r}=-[\vec{c}\;\vec{a}\;\vec{r}]\vec{b}-[\vec{b}\;\vec{c}\;\vec{r}]\vec{a}$

$\Rightarrow \quad [\vec{b}\;\vec{c}\;\vec{r}]\,\vec{a}+[\vec{c}\;\vec{a}\;\vec{r}]\vec{b}+[\vec{a}\;\vec{b}\;\vec{r}]\,\vec{c}=[\vec{a}\;\vec{b}\;\vec{c}]\,\vec{r}$

10. (b, c) : Given $P(A) = m$, $P(B) = p$ and $P(C) = c$,

$P(A\cup B\cup C)=0.75=\dfrac{3}{4}$

As A, B, C are independent.

$\therefore \quad m + p + c - mp - pc - mc + mpc = 3/4$ ...(i)

Probability of passing only two subjects is

$mp(1 - c) + pc(1 - m) + mc\,(1 - p) = 0.4=\dfrac{2}{5}$

$\Rightarrow \quad mp + pc + mc - 3mpc=\dfrac{2}{5}$ ...(ii)

Probability of passing atleast two subjects is

$mp(1 - c) + pc(1 - m) + mc(1 - p) + mcp = 0.5=\dfrac{1}{2}$

$\Rightarrow \quad mp + pc + mc - 2mcp = 1/2$ ...(iii)

Solving (ii) and (iii), we get $mcp=\dfrac{1}{2}-\dfrac{2}{5}=\dfrac{1}{10}$

And from (i) and (ii), we get $p+m+c=\dfrac{27}{20}$

11. (d) **12. (b)** **13. (a)**

We have, $f(x+y)-f(x-y)=f(x)[f(y)-f(-y)]$

Now, $f'(0)=\lim_{h\to0}\frac{f(h)-f(0)}{h}=\lim_{h\to0}\frac{f(h)-1}{h}=\log a$

Also, $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x-h)}{2h}$

$=\lim_{h\to0}\frac{f(x)\left[f(h)-f(-h)\right]}{2h}$

$=\lim_{h\to0}\frac{f(x)}{2}\left[\frac{f(h)-1}{h}+\frac{f(-h)-1}{-h}\right]=\frac{f(x)}{2}2\log a$

$\Rightarrow \frac{f'(x)}{f(x)}=\log a \Rightarrow \log f(x)=x\log a+c$

Now, $f(0) = 1 \Rightarrow c = 0 \Rightarrow f(x) = a^x$.

Hence $f'(x) = f(x)\log a = a^x \log a$

Now, $\frac{dy}{dx}=\frac{(x+y)^2}{(x+2)(y-2)} \Rightarrow \frac{dY}{dX}=\frac{(X+Y)^2}{XY}$

[Putting $x + 2 = X$, $y - 2 = Y$]

Put $Y=tX$ so that $t+X\frac{dt}{dX}=\frac{(1+t)^2}{t}$

$\Rightarrow dt-\frac{dt}{2t+1}=2\frac{dX}{X}$

$\Rightarrow \frac{y-2}{x+2}-\frac{1}{2}\ln\left|1+\frac{2(y-2)}{x+2}\right|=2\log(x+2)+c$

14. (P) → (4), (Q) → (1), (R) → (3)

(P) We have $x^2 + 1 = y$ and $y^2 + 1 = x$

On subtracting, we get $x^2 - y^2 = y - x \Rightarrow (x + y) = -1$ and $x \neq y$

On adding, we get $x^2 + y^2 + 2 = -1 \Rightarrow x^2 + y^2 + 3 = 0$ which again have no real roots.

Hence number of ordered pairs (x, y) is zero.

(Q) $\sin 2B=\frac{3}{2}\sin 2A$ and $3\sin^2 A=1-2\sin^2 B$

$\Rightarrow 3\sin^2 A = \cos 2B$

Now, $\cos(A + 2B) = \cos A \cos 2B - \sin A \sin 2B = 0$

$\therefore \sin(A + 2B) = 1$

(R) Equation of line passing through $(1, 0, -3)$ and parallel to given line is $\frac{x-1}{2}=\frac{y}{3}=\frac{z+3}{-6}$.

Coordinates of any point on the line are $P(2r + 1, 3r, -6r - 3)$. Putting this point in the equation of plane, we get $r = 1$

$\therefore$ Point P is $(3, 3, -9)$.

$\therefore$ Required distance $=\sqrt{2^2+3^2+(-6)^2}=7$

15. (0) : $a_k = 2a_{k-1} - a_{k-2}$

$\Rightarrow a_k - a_{k-1} = a_{k-1} - a_{k-2} = a_{k-2} - a_{k-3} = \ldots$

Thus the sequence $\{a_k\}$ is an A.P.

Also, $\frac{a_1^2+a_2^2+\ldots+a_{11}^2}{11}=90$

$\Rightarrow \frac{15^2+(15+d)^2+\ldots+(15+10d)^2}{11}=90$

$\Rightarrow 7d^2 + 30d + 27 = 0 \Rightarrow (7d + 9)(d + 3) = 0$

But $27 - 2a_2 > 0 \Rightarrow a_2 < 27/2 \therefore d = -3$

Now $\frac{a_1+a_2+\ldots+a_{11}}{11}=\frac{11}{2}\cdot\frac{\{30+10(-3)\}}{11}=0$

16. (0) : $\because 1 < x < 2 \Rightarrow [x] = 1$

$\Rightarrow f(x)=\cos\left(\frac{\pi}{2}-x^3\right)$

$\Rightarrow f'(x)=-\sin\left(\frac{\pi}{2}-x^3\right)\cdot(-3x^2) \therefore f'\left(\left(\frac{\pi}{2}\right)^{1/3}\right)=0$

17. (8) : Put $\ln x = t \Rightarrow dx = e^t\,dt$

$\therefore f(x)=\int_3^1\frac{3t^2+t^4}{(1+t^2-t^3)^2}dt$

$=\int\frac{\frac{t^4}{t^2}+\frac{t^2}{t^2}}{\left(\frac{1}{t^3}+\frac{1}{t}-1\right)^2}dt=-\int\frac{dz}{z^2}=\frac{1}{z}+C$

$\left[\text{Putting } \frac{1}{t^3}+\frac{1}{t}-1=z\right]$

$\Rightarrow f(x)=\frac{(\ln x)^3}{1+(\ln x)^2-(\ln x)^3}+C$

At $x = 1$, $f(1) = 0 + C \Rightarrow C = 0$

$\therefore f(x)=\frac{(\ln x)^3}{1+(\ln x)^2-(\ln x)^3} \Rightarrow |3f(e^2)| = 8$

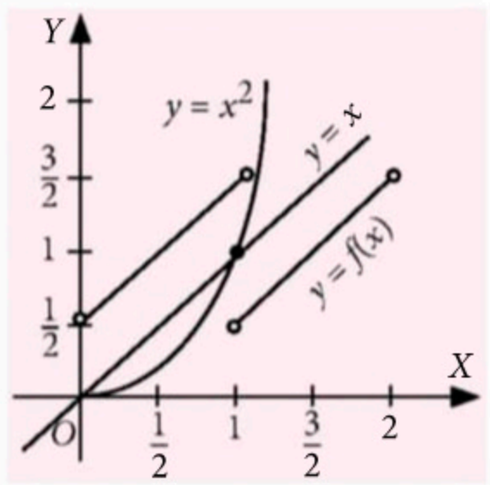
18. (3) : $f(x)=\{x\}+\frac{1}{2}$

$A=\int_0^1\left(x+\frac{1}{2}\right)dx+\int_1^2 x^2\,dx$

$=\left|\frac{x^2}{2}+\frac{x}{2}\right|_0^1+\left|\frac{x^3}{3}\right|_1^2$

$=1+\frac{8}{3}-\frac{1}{3}=\frac{10}{3}$

$\therefore [A] = 3$



OLYMPIAD CORNER

1. Determine the number of non-negative integers x that satisfy the equation $\left[\frac{x}{44}\right]=\left[\frac{x}{45}\right]$.

(Note : If r is any real number, then $[r]$ denotes the greatest integer less than or equal to r).

2. Draw a tangent line of parabola $y = x^2$ at the point $A(1, 1)$. Suppose the line intersects the x-axis and y-axis at D and B respectively. Let point C be on the parabola and point E on AC such that $\frac{AE}{EC} = \lambda_1$. Let point F be on BC such that $\frac{BF}{FC} = \lambda_2$ and $\lambda_1 + \lambda_2 = 1$. Assume that CD intersects EF at point P. When point C moves along the parabola, find the equation of the trail of P.

3. P is a point on the base BC of a triangle ABC. r, r_2, r_3 are the inradii of the triangles ABC, ACP, ABP respectively. h_a is the altitude of the triangle ABC from A. Prove that $\frac{1}{r_2}+\frac{1}{r_3}-\frac{r}{r_2r_3}=\frac{2}{h_a}$.

4. For non-negative reals x, y and z satisfying $x + y + z = 1$, show that $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right)\geq 64$.

5. The incircle of ABC touches BC, CA and AB at D, E and F respectively. X is a point inside ABC such that the incircle of XBC touches BC at D also, and touches CX and XB at Y and Z, respectively. Prove that $EFZY$ is a cyclic quadrilateral.

6. The real numbers a, b, c, x, y, z satisfy $a \geq b \geq c > 0$ and $x \geq y \geq z > 0$. Prove that $\frac{a^2x^2}{(by+cz)(bz+cy)}+\frac{b^2y^2}{(cz+ax)(cx+az)}+\frac{c^2z^2}{(ax+by)(ay+bx)}$ is at least 3/4.

7. Suppose that for any real x with $|x| \neq 1$, a function $f(x)$ satisfies $f\left(\frac{x-3}{x+1}\right)+f\left(\frac{3+x}{1-x}\right)=x$.

Find all possible $f(x)$.

SOLUTIONS

1. Let $\left[\frac{x}{44}\right]=\left[\frac{x}{45}\right]=n$

Since x is non-negative, n is also non-negative.

If $n = 0$, then x is any integer from 0 to $44 - 1 = 43$: a total of 44 values.

If $n = 1$, then x is any integer from 45 to $2 \times 44 - 1 = 87$: a total of 43 values.

If $n = 2$, then x is any integer from $2 \times 45 = 90$ to $3 \times 44 + 1 = 131$: a total of 42 values.

If $n = k$, then x is any integer from $45k$ to $44(k - 1) - 1 = 44k + 43$: a total of $(44k + 43) - (45k - 1) = 44 - k$ values.

Thus, increasing n by 1 decreases the number of values of x by 1. Also the largest value of n is 43, in which case x has only 1 value.

Therefore the number of non-negative integer values of x is $44+43+\ldots+1=\frac{1}{2}(44\times45)=990$.

2. The slope of the tangent line passing through A is $y' = [2x]_{x = 1} = 2$. So, the equation of the tangent line AB is $y = 2x - 1$

$\therefore$ Coordinates of B and D are AB is

$B(0, -1)$, $D\left(\frac{1}{2}, 0\right)$.

Thus D is the midpoint of AB.

Set $\gamma=\frac{CD}{CP}$, $t_1=\frac{CA}{CE}=1+\lambda_1$, $t_2=\frac{CB}{CF}=1+\lambda_2$.

Then $t_1 + t_2 = 3$.

Since AD is a median of ΔABC, $S_{\Delta CAB} = 2S_{\Delta CAD} = 2S_{\Delta CBD}$, where S_Δ denotes the area of Δ.

But $\frac{1}{t_1 t_2} = \frac{CE \cdot CF}{CA \cdot CB} = \frac{S_{\Delta CEF}}{S_{\Delta CAB}} = \frac{S_{\Delta CEP}}{2S_{\Delta CAD}} + \frac{S_{\Delta CFP}}{2S_{\Delta CED}}$

$= \frac{1}{2}\left(\frac{1}{t_1\gamma} + \frac{1}{t_2\gamma}\right) = \frac{t_1 + t_2}{2t_1 t_2 \gamma} = \frac{3}{2t_1 t_2 \gamma}$

So $\gamma = \frac{3}{2}$ and P is the center of gravity for ΔABC.

Consider $P(x, y)$ and $C(x_0, {x_0}^2)$. Since C is different from A, $x_0 \neq 1$. Thus the coordinates of the center of gravity P are $x = \frac{0+1+x_0}{3} = \frac{1+x_0}{3}$, $x \neq \frac{2}{3}$,

$y = \frac{-1+1+x_0^2}{3} = \frac{x_0^2}{3}$

Eliminating x_0, we get $y = \frac{1}{3}(3x-1)^2$

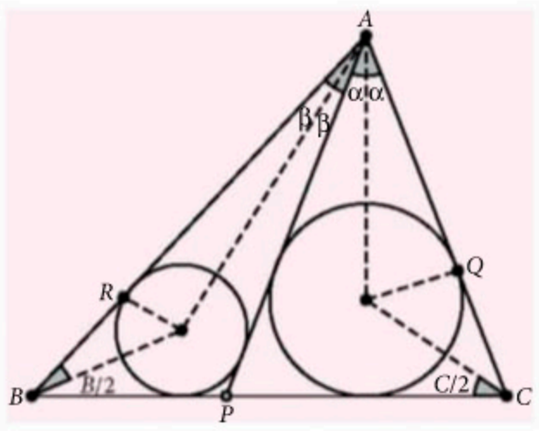
Thus the equation of the trail is

$y = \frac{1}{3}(3x-1)^2$, $x \neq \frac{2}{3}$

3. Let $BC = a$, $CA = b$, $AB = c$. Let α, β be the angles indicated in the figure.

We have, $\cot\alpha = \frac{AQ}{r_2} = \frac{AC - CQ}{r_2} = \frac{b}{r_2} - \cot\frac{C}{2}$

$\cot\beta = \frac{AR}{r_3} = \frac{AB - BR}{r_3}$



$= \frac{c}{r_3} - \cot\frac{B}{2}$

Also, $\cot\frac{A}{2} = \cot(\alpha+\beta)$

$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$

and hence $\cot\frac{A}{2}\left(\frac{b}{r_2} - \cot\frac{C}{2} + \frac{c}{r_3} - \cot\frac{B}{2}\right)$

$= \left(\frac{b}{r_2} - \cot\frac{C}{2}\right)\left(\frac{c}{r_3} - \cot\frac{B}{2}\right) - 1$

Rearranging, we get

$\frac{b}{r_2}\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right) + \frac{c}{r_3}\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right) - \frac{bc}{r_2 r_3}$

$= \cot\frac{A}{2}\cot\frac{B}{2} + \cot\frac{B}{2}\cot\frac{C}{2} + \cot\frac{C}{2}\cot\frac{A}{2} - 1$ ...(1)

Since $\cot\frac{A}{2} = \frac{s-a}{r}$, $\cot\frac{B}{2} = \frac{s-b}{r}$, $\cot\frac{C}{2} = \frac{s-c}{r}$

We have, $\left(\sum\cot\frac{A}{2}\cot\frac{B}{2}\right) - 1 = \sum\frac{(s-a)(s-b)}{r^2} - 1$

$= \frac{1}{r^2}(3s^2 - 2s(a+b+c) + ab + bc + ca) - 1$

$= \frac{1}{r^2}(ab + bc + ca - s^2) - 1$

Since area (Δ) of the triangle is given by

$\Delta = rs = \frac{abc}{4R} = \sqrt{s(s-a)(s-b)(s-c)}$

We have, $r^2 s^2 = s(s-a)(s-b)(s-c)$

$\Rightarrow r^2 s = s^3 - s^2(a+b+c) + s(ab+bc+ca) - abc$

$\Rightarrow \frac{1}{r^2}(ab + bc + ca - s^2) = \frac{4R\Delta}{r\Delta} + 1$

Hence, $\left(\sum\cot\frac{A}{2}\cot\frac{B}{2}\right) - 1 = \frac{4R}{r}$

Again $b = r\left(\cot\frac{A}{2} + \cot\frac{C}{2}\right)$, $c = r\left(\cot\frac{A}{2} + \cot\frac{B}{2}\right)$

Thus, from (1), we get $\frac{bc}{rr_2} + \frac{bc}{rr_3} - \frac{bc}{r_2 r_3} = \frac{4R}{r}$

This gives

$\frac{1}{r_2} + \frac{1}{r_3} - \frac{r}{r_2 r_3} = \frac{4R}{bc} = \frac{4Ra}{abc} = \frac{a}{\Delta} = \frac{a}{\frac{1}{2}a \cdot h_a} = \frac{2}{h_a}$

SAMURAI SUDOKU

ANSWER · FEBRUARY 2020

4	2	6	8	1	7	9	3	5				4	6	7	1	2	9	5	3	8
5	3	7	2	6	9	1	4	8				1	9	8	4	3	5	6	7	2
8	9	1	4	3	5	6	7	2				3	5	2	6	7	8	9	4	1
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4	1	9	2	7	5	8	6	3				5	6	3	9	1	2	7	4	8
5	3	2	4	8	6	7	9	1				4	9	8	6	5	7	1	2	3
8	7	6	9	1	3	4	5	2				7	2	1	3	8	4	9	5	6

4. Expanding, we first must show

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}\geq 64$$

By A.M.-G.M. inequality,

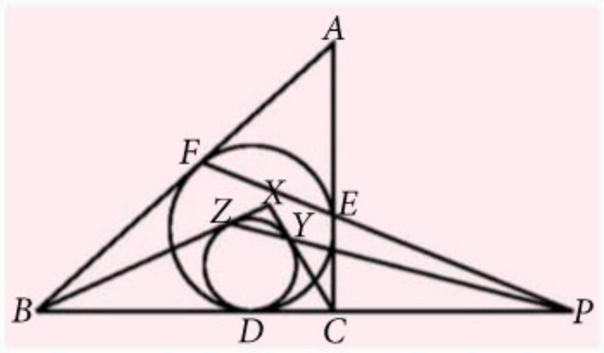
$$xyz\leq\left(\frac{x+y+z}{3}\right)^3=\frac{1}{27},\ \text{so that}\ \frac{1}{xyz}\geq 27.$$

$$\therefore\quad 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}$$

$$\geq 1+\frac{3}{\sqrt[3]{xyz}}+\frac{3}{\sqrt[3]{x^2y^2z^2}}+\frac{1}{xyz}$$

$$=\left(1+\frac{1}{\sqrt[3]{xyz}}\right)\geq 4^3=64.$$

5.



Let P be the intersection of EF with BC. Then by Menelaus' Theorem, we have

$$\frac{BP}{PC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FB}=1 \qquad ...(1)$$

Since $CE = CD$, $EA = AF$, and $FB = BD$, we get

$$\frac{BP}{PC}\cdot\frac{CD}{BD}=1\ \text{ so that }\ \frac{BP}{PC}=\frac{BD}{CD} \qquad ...(2)$$

Since $XZ = XY$, $BZ = BD$ and $CY = CD$, we have from (2)

$$\frac{BP}{PC}\cdot\frac{CY}{YX}\cdot\frac{XZ}{ZB}=\frac{BD}{CD}\cdot\frac{CD}{YX}\cdot\frac{XY}{BD}=1$$

Hence by Menelaus' Theorem P, Z and Y are collinear.
Since $PF\cdot PE = PD^2$ and $PZ\cdot PY = PD^2$, we have $PF\cdot PE = PZ\cdot PY$.
Hence, $EFZY$ is a cyclic quadrilateral.

6. Denote the left-hand side of the given inequality by S. Because $a\geq b\geq c$ and $x\geq y\geq z$ by the rearrangement inequality we have $bz + cy\leq by + cz$ so
$(by + cz)(bz + cy)\leq (by + cz)^2\leq 2((by)^2 + (cz)^2)$
Setting $\alpha = (ax)^2$, $\beta = (by)^2$, $\gamma = (cz)^2$, we obtain

$$\frac{a^2x^2}{(by+cz)(bz+cy)}\geq\frac{a^2x^2}{2\left((by)^2+(cz)^2\right)}=\frac{\alpha}{2(\beta+\gamma)}.$$

Adding to this the two analogous inequalities for the other summands, we find that

$$S\geq\frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right).$$

By the Cauchy-Schwarz inequality,

$$\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right)(\alpha(\beta+\gamma)+\beta(\gamma+\alpha)+\gamma(\alpha+\beta))$$

is at least $(\alpha + \beta + \gamma)^2$, which in turn equals

$$\frac{1}{2}\left((\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\right)+3(\alpha\beta+\beta\gamma+\gamma\alpha)$$

$$\geq\frac{3}{2}(2\alpha\beta+2\beta\gamma+2\gamma\alpha)$$

$$\therefore\quad S\geq\frac{1}{2}\left(\frac{\alpha}{\beta+\gamma}+\frac{\beta}{\gamma+\alpha}+\frac{\gamma}{\alpha+\beta}\right)\geq\frac{1}{2}\frac{(\alpha+\beta+\gamma)^2}{(2\alpha\beta+2\beta\gamma+2\gamma\alpha)}\geq\frac{3}{4}.$$

7. Set $t=\frac{x-3}{x+1}$, so that $x=\frac{t-3}{t+1}$ and $\frac{x-3}{x+1}=\frac{3+t}{1-t}$

Again we can rewrite the given equation, this time as

$$f\left(\frac{3+t}{1-t}\right)+f(t)=\frac{t-3}{t+1}$$

Adding these two equations, we have

$$\frac{8t}{1-t^2}=2f(t)+f\left(\frac{t-3}{t+1}\right)+f\left(\frac{3+t}{1-t}\right)=2f(t)+t,\ \text{so that}$$

$f(t)=\frac{4t}{1-t^2}-\frac{t}{2}$, and some algebra verifies that this solution works.

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.
The puzzle has a unique answer.

9								6
				2	9	3		8
2			6		7			
		8	9			6		
	6		2			5	7	
						1		
	4	6						
					8		3	
	2		1	3				7

	7	6	2					1
		3			4			8
9					5		2	7
			5					
					6		4	
	1			2			8	
			3		4		5	1
						3	6	
					8	7		

			2	9	1	6		
					8		1	
			4	1			6	
7		3		9				
	2		7			5		
8			4		1			2
	1				2			8
	3				5			
						6	1	
		7	3		4			5
	6					1		
1	4	9				2		

			3	7				
				9	1			
				4				
			6		5	7		3
				8		1		
					4			
5	6	1				8		2
	9					6		
8			7				9	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

Challenging PROBLEMS ON JEE Advanced

Exam on 17th May, 2020

Single Option Correct Type

1. Let $a_0 = a_1 = 1$ and for $n \geq 1, n(n+1)a_{n+1} = n(n-1)a_n - (n-2)a_{n-1}$, then $\lim_{n\to\infty} a_n =$

(a) 0 (b) 1 (c) e (d) ∞

2. Let a_n, b_n be positive integers and if $a_n + b_n\sqrt{2} = (2+\sqrt{2})^n$, where $n \in N$, then $\lim_{n\to\infty} \frac{a_n}{b_n} =$

(a) 2 (b) e^2 (c) $\sqrt{2}$ (d) $e^{\sqrt{2}}$

3. Which of the following statements is false?

(a) If $f : R \to R$ be continuous and periodic function with period T (> 0), then $\exists\ x_0 \in R$ such that $f\left(x_0 + \frac{T}{2}\right) = f(x_0)$.

(b) If $f(x)$ is continuous on $[0, 2]$ and $f(0) = f(2)$, then $\exists\ x_1, x_2$ in $[0, 2]$ such that $x_2 - x_1 = 1$ and $f(x_1) = f(x_2)$.

(c) If $f : [a, b] \to R$, $g : [a, b] \to R$, f and g are continuous functions on $[a, b]$ and $f(b) > g(b)$, $f(a) < g(a)$, then $\exists\ x_0 \in (a, b)$ such that $f(x_0) = g(x_0)$.

(d) If $f : [0, 1] \to [0, 1]$ be continuous function, then $f(x) \neq x$ for any $x \in [0, 1]$.

4. A real function f is always positive and satisfying $\frac{f(x)}{f(y)} \leq 2^{(x-y)^2}\ \forall\ x, y \in$ domain of f, then $f(x)$ can be

(a) $x^3 + \sqrt{x}$

(b) $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$

(c) $\int_0^x 2t^5 dt$ (d) $21x^2 + 5x^3 + 19x + 78$

5. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$, then a possible value of α^2 is

(a) 4 (b) 3 (c) 2 (d) 1

6. If f is a twice differentiable function over the positive reals such that $f(x) + 2x^3 f'(x) + x^4 f''(x) = 0$ for all positive reals x and $\lim_{x\to\infty} x f(x) = 1$. The real number λ such that $f(\lambda) = 1/2$ is

(a) $5/\pi$ (b) $6/\pi$ (c) $7/\pi$ (d) $8/\pi$

7. Let $f(x) = x^3 - \frac{3}{2}x^2 + x + \frac{1}{4}$. Then,

$$\int_0^1 \underbrace{f(f(f(f(\ldots.f}_{21\text{ times}}(x))))) dx =$$

(a) 0 (b) 1 (c) 1/2 (d) 1/3

8. Suppose $p(x)$ is a polynomial such that $p(1) = 1$ and $\frac{p(2x)}{p(x+1)} = 8 - \frac{56}{x+7}$ for $x \in R$ for which both sides are defined. Then $|p(-1)| =$

(a) $\frac{5}{21}$ (b) $\frac{21}{5}$ (c) $\frac{16}{5}$ (d) $\frac{5}{16}$

9. The maximum value of $\frac{x^4 - x^2}{x^6 + 2x^3 - 1}$ for $x > 1$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{18}$

Comprehension Type

Paragraph for Q. No. 10 and 11

Consider the following matrices P, Q as given below.

$$P = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix},\ Q = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

By : Tapas Kr. Yogi (Prime Axis Classes), Visakhapatnam Mob : 09533632105, 09949428844

10. The value of det $(P + P^2Q^2 + P^3 + P^4Q^4 + \ldots.$ (1000 terms)) =

(a) 0 (b) −500 (c) 500 (d) 1000

11. If $Q^{-1} = Q^2 - kQ - I_3$, $k \in R$, then k =

(a) −1 (b) 1 (c) 2 (d) −2

Numerical Value Type

12. Given a point (2, 1). If the minimum perimeter of triangle with one vertex at (2, 1), one on the x-axis and one on the line $y = x$ is k then $[k]$ equals (where $[\cdot]$ denotes greatest integer function)

13. Let ABC be a given triangle with $AB = 3$, $AC = 4$, $BC = 5$. Median and altitude through A cuts side BC at D, E and circumcircle at P, Q respectively, then the value of $\frac{25}{4} \cdot \frac{EQ}{DP} =$

14. The function $f : R \to R$ satisfies $f(x^2)f''(x) = f'(x)f'(x^2)$ for all x. Given that $f(1) = 1$ and $f'''(1) = 8$, then $f'(1) + f''(1) =$

15. Vertices of a parallelogram $ABCD$ are $A(3, 1)$, $B(13, 6)$, $C(13, 21)$ and $D(3, 16)$. A line passing through origin, divides the parallelogram into two congruent parts. Slope of such line is $\frac{a}{b}$ in lowest terms, where $|a - b| =$

16. The area of the region in XY plane satisfying $x^6 - x^2 + y^2 \le 0$ is $\frac{\pi}{k}$, where $k =$

17. A is a $n \times n$ matrix whose all elements are 1 and B is a $n \times n$ matrix whose diagonal elements are all n and other elements are $(n - r)$, then $|(B - rI)(B - (n^2 - nr + r)I)| =$

SOLUTIONS

1. (a) : Notice that $a_n = \frac{1}{n!}$

2. (c) : $a_n + b_n\sqrt{2} = (2+\sqrt{2})^n$ and

$$a_n - b_n\sqrt{2} = (2-\sqrt{2})^n$$

Hence, required limit

$$= \sqrt{2} \lim_{n\to\infty} \left[\frac{(2+\sqrt{2})^n + (2-\sqrt{2})^n}{(2+\sqrt{2})^n - (2-\sqrt{2})^n} \right] = \sqrt{2}$$

3. (d) : We take examples for proving these particular statements, like,

(a) $g(x) = f\left(x + \frac{T}{2}\right) - f(x)$

(b) $h(x) = f(x + 1) - f(x)$

(c) $h(x) = f(x) - g(x)$ (d) $g(x) = f(x) - x$

4. (b) : Taking log on both sides, we get

$$\log_2 f(x) - \log_2 f(y) \le (x - y)^2$$

Putting $y = x + h$, $-(\log_2 f(x))' \le 0$ as $h \to 0$

$\Rightarrow$ $(\log_2(f(x)))' \ge 0$

Putting $x = y + h$, $(\log_2 f(y))' \le 0$ as $h \to 0$

$\Rightarrow$ $(\log_2(f(x)))' = 0 \Rightarrow f'(x) = 0$

i.e., $f(x)$ is constant function.

5. (c) : For the given condition to hold, the point $(2\alpha, \alpha)$ should lie inside the director circle and outside the ellipse. Simplification gives $\alpha^2 \in \left(\frac{9}{7}, \frac{12}{5}\right)$.

6. (b) : Replacing x by $1/y$, we have

$$f\left(\frac{1}{y}\right) + \frac{2}{y^3} f'\left(\frac{1}{y}\right) + \frac{1}{y^4} f''\left(\frac{1}{y}\right) = 0$$

$$\Rightarrow f\left(\frac{1}{y}\right) + \frac{d^2}{dy^2}\left(f\left(\frac{1}{y}\right)\right) = 0$$

$$\Rightarrow f\left(\frac{1}{y}\right) = c_1 \cos y + c_2 \sin y \text{ (Standard S.H.M. eqn.)}$$

$$\text{i.e., } f(x) = c_1 \cos\left(\frac{1}{x}\right) + c_2 \sin\left(\frac{1}{x}\right)$$

Form IV

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I, Mahabir Singh, authorised signatory for MTG Learning Media Pvt. Ltd. hereby declare that particulars given above are true to the best of my knowledge and belief.

For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

CONCEPT MAP

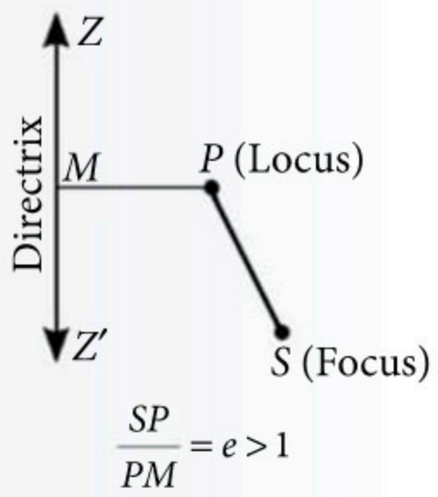
HYPERBOLA

Class XI

Hyperbola

Definition

locus of a point which moves
the plane in such a way
at the ratio of its distance
om a fixed point (focus) to
di nce from a fixed line
irectrix) is a constant greater
an unity. The constant
tio is denoted by e, called
centricity.



$$\frac{SP}{PM} = e > 1$$

Conjugate Hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate perbola is given by $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

Rectangular Hyperbola

hyperbola is said to be rectangular, if the asymptotes e at right angle. Equation of rectangular hyperbola $x^2 - y^2 = a^2$ or $xy = c^2$.

Basic Terms

$= c^2$ is a rectangular hyperbola, then

rtices are $(c, c), (-c, c)$.

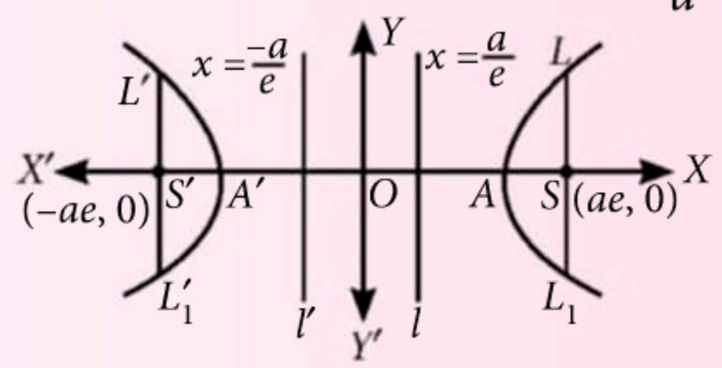
ci are $(\pm c\sqrt{2}, \pm c\sqrt{2})$.

uations of directrices are $x + y \pm c\sqrt{2} = 0$.

uation of transverse axis is $y = x$.

General Terms

Standard equation of hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.



- $O(0, 0)$ is the centre of hyperbola.
- $S(ae, 0)$, $S'(-ae, 0)$ are foci of hyperbola.
- $A(a, 0)$, $A'(-a, 0)$ are the vertices.
- Equation of directrices are $x = \frac{a}{e}$ and $x = -\frac{a}{e}$.
- Length of transverse axis = $2a$
- Length of conjugate axis = $2b$
- Length of latus rectum, $LL_1 = L'L_1' = \frac{2b^2}{a}$
- Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Some Important Results

If $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$, $T \equiv \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$, $S_1 \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$, then

- Location of $P(x_1, y_1)$: P lies inside the hyperbola $S = 0$ iff $S_1 > 0$. P lies outside the hyperbola $S = 0$ iff $S_1 < 0$ and P lies on the hyperbola $S = 0$ iff $S_1 = 0$.
- Tangent to $S = 0$ at $P(x_1, y_1)$: $T = 0$
- Tangents to $S = 0$ with slope m are $y = mx \pm \sqrt{a^2m^2 - b^2}$
- Pair of tangents to $S = 0$ from $P(x_1, y_1)$: $T^2 = SS_1$
- Normal to $S = 0$ at $P(x_1, y_1)$: $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$
- Director circle of $S = 0$ is $x^2 + y^2 = a^2 - b^2$.
- Auxiliary circle of $S = 0$ is $x^2 + y^2 = a^2$.

PROBABILITY

Class XII

CONCEP
MAP

Probability

Conditional Probability

Probability of occurrence of an event A, given that B has already happened is called conditional probability

$$i.e., P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

- $0 \leq P(A/B) \leq 1$
- $P(A'/B) = 1 - P(A/B)$
- $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$, where $P(F) \neq 0$

Probability Distribution

- Let real numbers $x_1, x_2, ..., x_n$ are the possible values of a random variable X and $p_1, p_2, ..., p_n$ are the corresponding probabilities to each value of the random variable X. Then, the probability distribution is

X	:	x_1	x_2	...	x_n
$P(X)$	:	p_1	p_2	...	p_n

Note : (i) $p_i > 0$

(ii) $\sum_{i=1}^{n} p_i = 1$

- If X is a random variable, then

(i) Mean, $\bar{X} = E(X) = \mu = \sum_{i=1}^{n} p_i x_i$

(ii) Variance (X), σ^2

$$= \sum_{i=1}^{n} p_i x_i^2 - \left(\sum_{i=1}^{n} p_i x_i \right)^2$$

(iii) Standard deviation, σ

$= \sqrt{\text{Var}(X)}$

Events

Independent

Two events A and B in a sample space S are said to be independent if

- $P(A \cap B) = P(A) \cdot P(B)$
- $P(A/B) = P(A)$
- $P(B/A) = P(B)$

No : If A and B are independent events, then A and B' ; A' and B ; A' and B' are also independent.

Mutually Exclusive

- Two events A and B in a sample space S are said to be mutually exclusive if and only if $A \cap B = \phi$.

Exhaustive

- Two events A and B in a sample space S are said to be exhaustive if $A \cup B = S$.

Binomial Distribution

- In a series of n independent trials, if the probability of success (p) in each trial is same, then the probability of r successes is $P[X = r] = \binom{n}{r} p^r \cdot q^{n-r}$, $q = 1 - p$, where q is the probability of failure.
- Mean : np
- Variance : npq
- Standard deviation : $\sqrt{npq}$

Theorems

- **Multiplication Theorem :**
  $P(A \cap B) = P(A) \cdot P(B / A)$
  $= P(B) \cdot P(A / B)$, provided $P(A) \neq 0$, P

- **Total Probability Theorem :** $P($
  $P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) +$
  $+ P(E_n)\ P(A/E_n)$, where E_1, E
  ....., E_n are mutually exclusive
  exhaustive events of sample spa

Bayes' Theorem

- $$P(E_i / A) = \frac{P(E_i)\ P(A / E_i)}{\sum_{i=1}^{n} P(E_i)\ P(A / E_i)}$$

where E_1, E_2, ..., E_n are mut
exclusive and exhaustive even
sample space S.

Bernoulli Trials

Set of n trials is said to be Bernoull

- the value of n is finite.
- each and every trial/experime independent.
- trial (experiment) consist of two outcomes namely success failure.
- probability of success and failur each trial is fixed (same).

$\lim_{x\to\infty} xf(x)=1$ gives $c_1 = 0, c_2 = 1$

So, $f(\lambda)=\sin\left(\frac{1}{\lambda}\right)=\frac{1}{2} \Rightarrow \lambda=\frac{6}{\pi}$

7. (c) : $f(x)=\frac{1}{2}+\frac{1}{4}[x^4-(1-x)^4]$

Notice that $f(x) + f(1 - x) = 1 \Rightarrow f(1 - x) = 1 - f(x)$

By induction,

$$\underbrace{f(f(f(f\ldots f}_{n\text{ times}}(1-x))))=1-\underbrace{f(f(f(f\ldots.f}_{n\text{ times}}(x))))$$

So, given integral, $I=\int_0^1[1-\underbrace{f(f(f\ldots f}_{21\text{ times}}(1-x)))]\,dx$

$$=1-\int_0^1 \underbrace{f(f(f\ldots f}_{21\text{ times}}(1-x)))\,dx$$

$$=1-\int_0^1 \underbrace{f(f\ldots f}_{21\text{ times}}(t))\,dt \text{ [Putting } 1-x=t]$$

$= 1 - I \Rightarrow 2I = 1 \Rightarrow I = 1/2$

8. (a) : Rewriting the equation as

$(x + 7)\, p(2x) = 8x\, p(x + 1)$

Equating degrees on both sides, $a_n \cdot 2^n = a_n \cdot 8$ where a_n is the leading term gives $n = 3$, *i.e.*, $p(x)$ is a cubic polynomial.

Also, $p(0) = 0 = p(-2) = p(-6)$

gives $p(x)=\frac{1}{21}\cdot x(x+2)(x+6)$

9. (b) : Given expression can be written as,

$$E=\frac{\left(x-\frac{1}{x}\right)}{\left(x-\frac{1}{x}\right)^3+2+3\left(x-\frac{1}{x}\right)} \text{ and}$$

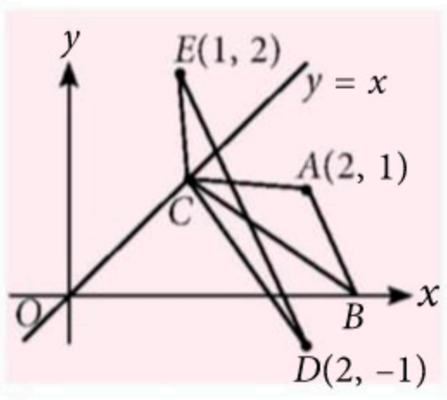
$\left(x-\frac{1}{x}\right)^3+1+1\geq 3\left(x-\frac{1}{x}\right)$ (By A.M. – G.M. inequality)

$$\Rightarrow E\leq\frac{\left(x-\frac{1}{x}\right)}{3\left(x-\frac{1}{x}\right)+3\left(x-\frac{1}{x}\right)} \Rightarrow E\leq\frac{1}{6}.$$

10. (a) **11. (a)**

Notice that $|P| = 0$, $|Q| = 1$,
$P^2 = P$ and $Q^2 = I$

12. (3) : Let $A(2, 1)$, $B(\alpha, 0)$ on x-axis and $C(h, h)$ on $y = x$, then let $D(2, -1)$ be the reflection of A in x-axis and $E(1, 2)$ be the reflection of A in $y = x$ line. Then AB



$= BD$ and $AC = CE$

So, perimeter of $\Delta ABC = AB + BC + AC$

$= BD + BC + CE > CD + CE > DE = \sqrt{10}$

Hence, $k=\sqrt{10}$.

13. (6) : For median, we have

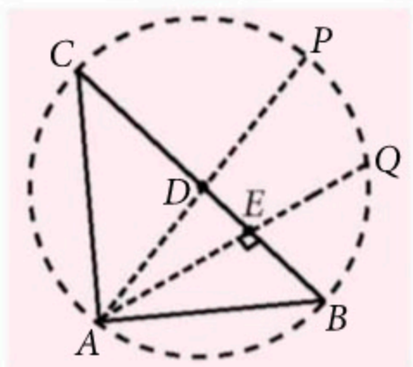
$AD \cdot DP = BD \cdot CD$

$\Rightarrow \frac{5}{2}\cdot DP=\frac{5}{2}\cdot\frac{5}{2}$

and for altitude, we have

$AE \cdot EQ = BE \cdot CE$

$\Rightarrow \frac{12}{5}\cdot EQ=\frac{9}{5}\cdot\frac{16}{5}$



14. (6) : Let $f'(1) = t$, $f''(1) = y$, then

Putting $x = 1$ in the given equation, we get $y = t^2$, and differentiating the given equation and then again putting $x = 1$, we get $ty = 8$. Hence, $t = 2$, $y = 4$.

Note that such a function can be $f(x) = e^{2x - 2}$.

15. (3) : Note that the line passes through the middle of the parallelogram, *i.e.*, (8, 11)

16. (2) : $|y|\leq\sqrt{x^2-x^6}$

Taking symmetry into account, we get

$$\text{Required area} = 4\cdot\int_0^1\sqrt{x^2-x^6}\,dx=\frac{\pi}{2}$$

17. (0) : Notice that $A^2 = nA$ and $B - rI = (n - r)A$

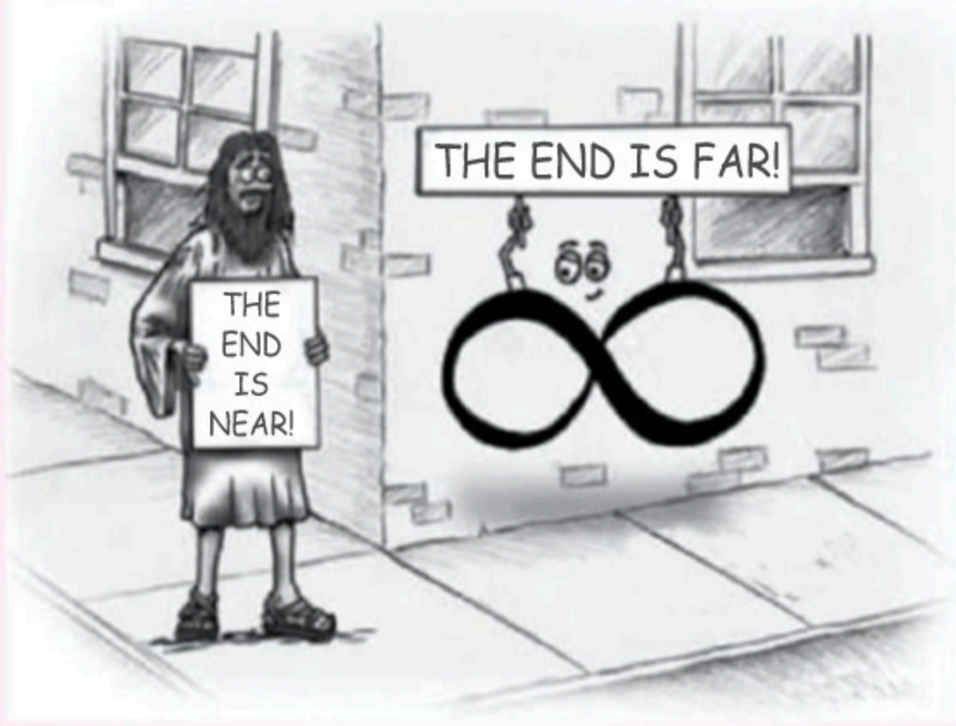
So, required determinant becomes

$|(B - rI)(B - rI - n(n - r)I)|$

$= |(n - r)A((n - r)A - n(n - r)I)|$

$= (n - r)^2\,|nA - nA| = 0$

COMIC CAPSULE



MATHS MUSING

SOLUTION SET-206

1. (c) : Given, a, b, c are in H.P. $\Rightarrow b = 2ac/(a+c)$ ...(i)

b, c, d are in G.P. $\Rightarrow c^2 = bd$ ...(ii)

and c, d, e are in A.P. $\Rightarrow d = (c+e)/2$ ...(iii)

From (i), $ab + bc = 2ac \Rightarrow c = ab/(2a-b)$ ...(iv)

Also, $d = \frac{1}{2}\left(\frac{ab}{2a-b} + e\right)$ [Using (iii) and (iv)] ...(v)

On putting the values of c and d from (iv) and (v) in (ii), we get

$$\frac{a^2b^2}{(2a-b)^2} = \frac{b}{2}\left[\frac{ab}{2a-b} + e\right] \Rightarrow e = \frac{ab^2}{(2a-b)^2}$$

2. (d): Let $z = r_1e^{i\theta}$ and $w = r_2e^{i\phi} \Rightarrow \bar{z} = r_1e^{-i\theta}$

Given, $|zw| = 1 \Rightarrow |r_1e^{i\theta}\cdot r_2e^{i\phi}| = 1 \Rightarrow r_1r_2 = 1$ ...(i)

and $\arg(z) - \arg(w) = \frac{\pi}{2} \Rightarrow \theta - \phi = \frac{\pi}{2}$ ...(ii)

Now, $\bar{z}w = r_1e^{-i\theta}\cdot r_2e^{i\phi} = r_1r_2e^{-i(\theta-\phi)} = 1\cdot e^{-\pi/2}$

[From (i) and (ii)]

$$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$$

3. (c) : $T_{r+1} = {}^{6561}C_r\,(7^{1/3})^{6561-r}\cdot(11^{1/9})^r$

$$= {}^{6561}C_r\cdot 7^{2187-\frac{r}{3}}\cdot 11^{\frac{r}{9}}$$

T_{r+1} is rational, if $\frac{r}{9}$ and $\frac{r}{3}$ are integers. So, r is a multiple of 9.

$\because\ 0 \le r \le 6561 \Rightarrow 0 \le \frac{r}{9} \le 729 \Rightarrow \frac{r}{9} = 0, 1, 2, 3, \ldots, 729$

$\therefore$ Total terms = 730

4. (a) : The number of 5-digit numbers

$= 6 \times 6 \times 5 \times 4 \times 3 = 2160$

The possible ways of choosing 5 digits for getting number divisible by 3 are (1, 3, 4, 6, 7), (0, 1, 2, 3, 6), (0, 1, 3, 4, 7), (0, 1, 4, 6, 7), (0, 2, 3, 6, 7), (0, 2, 3, 4, 6)

The number of numbers using them = 5! + 5 (5! – 4!)

= 120 + 5 × 96 = 600

$\therefore$ Required probability $= \frac{600}{2160} = \frac{5}{18}$

5. (b): Replacing $f(x)$ by x and x by $g(x)$.

$x = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$. Differentiating, $1 = \frac{g'}{\sqrt{1+g^3}}$

or $(g')^2 = 1 + g^3$. Differentiating, $g'' = \frac{3}{2}g^2$.

6. (a, b, c) : $\vec{a} = p(\vec{x}\times(\vec{y}\times\vec{z}))$, where $p \in R - \{0\}$

$= p\{(\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}\} = p(\vec{y} - \vec{z})$

Now, $\vec{a}\cdot\vec{y} = p\{\vec{y}\cdot\vec{y} - \vec{y}\cdot\vec{z}\} = p\{2-1\} = p$

$\therefore\ p = \vec{a}\cdot\vec{y}$

Thus, $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y} - \vec{z})$ ...(i)

Similarly, $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z} - \vec{x})$ ...(ii)

Now, $\vec{a}\cdot\vec{b} = (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})\{(\vec{y} - \vec{z})\cdot(\vec{z} - \vec{x})\}$ [From (i) and (ii)]

$= (\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})(\vec{y}\cdot\vec{z} - \vec{y}\cdot\vec{x} - \vec{z}\cdot\vec{z} + \vec{z}\cdot\vec{x}) = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$

7. (d): $\because$ Both lines are coplanar.

$$\therefore \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

$\therefore\ \sin^{-1}(\sin 4) = \sin^{-1}(\sin(\pi - 4)) = \pi - 4$

8. (d): Let $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{4} = r_1$

$\Rightarrow\ x = 3 + 2r_1,\ y = 2 + 3r_1,\ z = 1 + 4r_1$

It will lie on $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$

$\Rightarrow \frac{3 + 2r_1 - 2}{3} = \frac{2 + 3r_1 - 3}{2} \Rightarrow r_1 = 1$

So, point of intersection is (5, 5, 5).

9. (1.92) : Since, we are given rate per rupees, harmonic mean will give the correct answer.

$$\text{H.M.} = \frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \frac{4\times 12}{25} = 1.92 \text{ kg per rupee}$$

10. (c) : P → 4, Q → 1, R → 2, S → 3

(P) z_1, z_2, z_3 are the vertices of an equilateral triangle if $z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$.

(Q) Since, $\frac{z_3 - z_1}{z_3 - z_2}$ is purely imaginary, the triangle is right angled with right angle at z_3.

(R) It is a obtuse angled triangle.

(S) $\frac{z_3 - z_1}{z_3 - z_2}$ is purely imaginary, the triangle is right angled and isosceles.

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CONCEPT BOOSTER

Probability

*ALOK KUMAR

RANDOM EXPERIMENT

- An experiment, whose all possible outcomes are known in advance but the outcome of any specific performance can not be predicted before the completion of the experiment, is known as random experiment. For example : Tossing of a coin.

Sample Space

- A set of all possible outcomes associated with same random experiment is called sample space and denoted by S. For instance, experiment of rolling a die, then sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Event

- An event is a subset of sample space.

Simple Event

- If an event is a set containing only one element of the sample space, then it is called a simple event.

Compound Event

- A compound event is one that can be represented as a union of sample points. For instance, the event of drawing a heart from a deck of cards is the subset $A = \{\text{heart}\}$ of the sample space $S = \{\text{heart, spade, club, diamond}\}$. Therefore A is a simple event. Now the event B of drawing a red card is a compound event since $B = \{\text{heart} \cup \text{diamond}\} = \{\text{heart, diamond}\}$.

PROBABILITY

- If a random experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes are favourable to A, then the probability of event A, $P(A) = \dfrac{\text{favourable cases}}{\text{total number of cases}} = \dfrac{n}{N}$.
- If the probability of certain event is one, it doesn't mean that event is going to happen with certainty. Infact we would be just predicting that, the event is most likely to occur in comparison to other events. Predictions depend upon the past information and of course also on the way of analysing the information at hand.
- Similarly if the probability of certain event is zero, it doesn't mean that the event can never occur.

Mutually Exclusive Events

- A set of events is said to be mutually exclusive if the occurrence of one of them precludes the occurrence of any of the other events. For instance, when a pair of dice is tossed, the events 'a sum of 4 occurs', 'a sum of 10 occurs' and 'a sum of 12 occurs' are mutually exclusive. Using set theoretic notation, if $A_1, A_2, .., A_n$ be the set of mutually exclusive events, then $A_i \cap A_j = \phi$ for $i \neq j$ and $1 \leq i, j \leq n$.

Exhaustive Events

- A set of events is said to be exhaustive if the performance of random experiment always results in the occurrence of at least one of them. For instance, consider an ordinary pack of cards, then the events 'drawn card is heart', 'drawn card is diamond', 'drawn card is club' and 'drawn card is spade' is set of exhaustive event.
- If 'E' be an exhaustive event, then $P(E) = 1$.

Some Important Points

If 'A' and 'B' be any two events of the sample space, then

- $A \cup B$ stands for occurrence of at least one of them.
- $A \cap B$ stands for simultaneous occurrence of A and B.

- $\bar{A}$ (or A') stands for non-occurrence of A.
- $\bar{A} \cap \bar{B}$ (or $A' \cap B'$) stands for non-occurrence of both A and B.
- $A \subseteq B$ stands for 'the occurrence of A implies the occurrence of B'.
- If A and B are any two events, then
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive events, then
  $P(A \cup B) = P(A) + P(B)$.
- $P(A') = 1 - P(A)$
- $P(A \cap B') = P(A) - P(A \cap B)$,
  $P(A' \cap B) = P(B) - P(A \cap B)$
- $P(A' \cup B') = 1 - P(A \cap B)$,
  $P(A' \cap B') = 1 - P(A \cup B)$
- P(exactly one of A, B occurs)
  $= P(A \cap B') + P(A' \cap B) = P(A) + P(B) - 2P(A \cap B)$
  $= P(A \cup B) - P(A \cap B) = P(A' \cup B') - P(A' \cap B')$

If A, B, C are any three events of the sample space, then

- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
  $- P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
- P (exactly one of A, B, C occurs)
  $= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(A \cap C)$
  $- 2P(B \cap C) + 3P(A \cap B \cap C)$
- P(exactly two of A, B, C occurs) $= P(A \cap B)$
  $+ P(B \cap C) + P(A \cap C) - 3P(A \cap B \cap C)$
- P (at least two of A, B, C occurs) $= P(A \cap B)$
  $+ P(B \cap C) + P(A \cap C) - 2P(A \cap B \cap C)$
- If $A_1, A_2, ..., A_n$ are n events, then $P\left(\bigcup_{i=1}^{n} A_i\right)$

$$= \sum_{r=1}^{n} P(A_r) - \sum_{1 \le r_1 < r_2 \le n} (P(A_{r_1} \cap A_{r_2}) +$$

$$\sum_{1 \le r_1 < r_2 < r_3 \le n} P(A_{r_1} \cap A_{r_2} \cap A_{r_3}) - ... + (-1)^{n-1} \; P\left(\bigcap_{i=1}^{n} A_i\right)$$

- $P(A \cup B) \ge \max(P(A), P(B), P(A) + P(B) - 1)$
- If out of $m + n$ equally likely, mutually exclusive and exhaustive cases, m cases are favourable to an event A and n are not favourable to event A, then $m : n$ is called odds in favour of A and $n : m$ is called odds against the event A.

CONDITIONAL PROBABILITY

- The probability of occurrence of an event B when it is known that some event A has already occurred is called conditional probability and is denoted by $P(B/A)$ and defined as :

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \; 0 < P(A) \le 1$$

Similarly, $P(A/B) = \dfrac{P(A \cap B)}{P(B)}, \; 0 < P(B) \le 1$

Note : The symbol $P(B/A)$ is read as "probability of B, given A".

INDEPENDENT EVENTS

- Two events A and B are said to be independent if occurrence or non-occurrence of one does not affect the occurrence or non-occurrence of the other,

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

$\Rightarrow \quad P(B/A) = \dfrac{P(A \cap B)}{P(A)} = P(B)$

$\therefore \quad P(B/A) = P(B), \; P(A) \ne 0$

Similarly, $P(A/B) = P(A), \; P(B) \ne 0$

If the events are not independent, they are said to be dependent.

- Distinction between independent and mutually exclusive events must be carefully made. Independence is a property of probability whereas the mutual exclusion is a set-theoretic concept. If A and B are two mutually exclusive and possible events of sample space 'S' then $P(A) > 0$, $P(B) > 0$ and $P(A \cap B) = 0 \ne P(A) \cdot P(B)$ so that A and B can't be independent. Infact $P(A/B) = 0$, similarly $P(B/A) = 0$. Consequently, mutually exclusive events are strongly dependent.
- Two events A and B are independent if and only if A and B' are independent or A' and B are independent or A' and B' are independent.

We have, $P(A \cap B) = P(A) \cdot P(B)$

Now, $P(A \cap B') = P(A) - P(A \cap B)$

$= P(A) - P(A) \cdot P(B) = P(A)\,(1 - P(B)) = P(A) \cdot P(B')$

Thus, A and B' are independent.

Similarly, $P(A' \cap B) = P(B) - P(A \cap B)$

$= P(B) \cdot P(A')$

Thus, B and A' are independent.

Finally $P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - P(A) - P(B) + P(A) \cdot P(B)$

$= (1 - P(A))\,(1 - P(B)) = P(A') \cdot P(B')$

Thus, A' and B' are also independent.

TOTAL PROBABILITY THEOREM AND BAYES' THEOREM

- **Partition of Sample Space:** Consider a sample space 'S'. Let $A_1, A_2, ..., A_n$ be the set of mutually exclusive and exhaustive set of sample space S.

These $A_1, A_2, ..., A_n$ events are said to partition of the sample space into 'n' parts.

$\therefore \quad A_i \cap A_j \neq \phi$ for $i \neq j$, $1 \leq i, j \leq n$

And $\sum_{i=1}^{n} P(A_i) = 1$

Total Probability Theorem

- Let 'A' be any event of S. We can write
  $A = (A_1 \cap A) \cup (A_2 \cap A) \cup ... \cup (A_n \cap A)$.
  As $A_1, A_2, ..., A_n$ are mutually exclusive, $(A_1 \cap A)$, $(A_2 \cap A)$, ..., $(A_n \cap A)$ would also be mutually exclusive.

$$\therefore \quad P(A) = P(A_1 \cap A) + P(A_2 \cap A) + ... + P(A_n \cap A)$$
$$= P(A_1)\cdot P(A/A_1) + P(A_2)\cdot P(A/A_2) + ... + P(A_n)\cdot P(A/A_n)$$

$$\Rightarrow \quad P(A) = \sum_{i=1}^{n} P(A_i)\cdot P(A/A_i)$$

This is known as the total probability of the event A.

Bayes' Theorem

- Let us consider any event 'A' of sample space 'S'. This event would have occurred due to the occurrence of any of the events $A_1, A_2, ..., A_n$.

Now, let us say that event A is found to have occurred and we have to find the probability that it has occurred to the occurrence of cause, say A_i. That means we are interested in $P(A_i/A)$. These types of problems are solved with the help of Bayes' theorem.

From total probability theorem, we get

$$P(A) = \sum_{i=1}^{n} P(A_i)\cdot P(A/A_i)$$

Also, $P(A_i/A) = \dfrac{P(A_i \cap A)}{P(A)} = \dfrac{P(A_i)\cdot P(A/A_i)}{P(A)}$

$$\Rightarrow \quad P(A_i/A) = \frac{P(A_i)\cdot P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\cdot P(A/A_i)}$$

This result is known as Bayes' theorem.

BINOMIAL TRIALS AND BINOMIAL DISTRIBUTION

- Consider a random experiment whose outcomes can be classified as success or failure. It means that experiment results in only two outcomes E_1(success) or E_2 (failure). Further assume that experiment can be repeated several times, probability of success or failure in any trial are p and q ($p + q = 1$) and don't vary from trial to trial and finally different trials are independent. Such an experiment is called binomial experiment and trials are said to be binomial trials.
- A probability distribution representing the binomial trials is said to be binomial distribution.
- Let us consider a binomial experiment which has been repeated 'n' times. Let the probability of success and failure in any trial be p and q respectively and we are interested in the probability of occurrence of exactly 'r' successes in these n trials. Now number of ways of choosing 'r' successes in 'n' trials = ${}^nC_r$. Probability of 'r' successes and $(n - r)$ failures is $p^r \cdot q^{n-r}$. Thus, probability of having exactly 'r' successes = ${}^nC_r \cdot p^r \cdot q^{n-r}$
- Let 'X' be a random variable representing the number of successes, then

$$P(X = r) = {}^nC_r \cdot p^r \cdot q^{n-r} \quad (r = 0, 1, 2, ..., n)$$
$$1 = (p + q)^n = {}^nC_0\, p^0 q^n + {}^nC_1\, p^1 q^{n-1} + {}^nC_2\, p^2 q^{n-2} + ... + {}^nC_r\, p^r q^{n-r} + ... + {}^nC_n\, p^n q^0$$

$X \rightarrow$ Number of success 0, 1, 2, ..., r, ..., n

- Probability of at most 'r' successes in n trials
$$= \sum_{r=0}^{r} {}^nC_r p^r q^{n-r}$$
- Probability of at least 'r' successes in n trials
$$= \sum_{r=r}^{n} {}^nC_r p^r \cdot q^{n-r}$$
- Probability of having 1st success at the r^{th} trial
$$= p \cdot q^{r-1}$$
- Mean = np, Variance = npq,
  Standard deviation $= \sqrt{npq}$

PROBLEMS

Single Correct Answer Type

1. A number of six digits is written down at random. Probability that sum of digits of the number is even, is

(a) 1/2 (b) 3/8
(c) 3/7 (d) none of these

2. Two small squares on a chess board are chosen at random. Probability that they have a common side is

(a) 1/3 (b) 1/9 (c) 1/18 (d) 1/8

3. A bag contains 100 balls numbered from 1 to 100. Four balls are removed at random without replacement. The probability that the number on the last ball is smaller than the number on first ball, is equal to

(a) 1/4 (b) 3/4 (c) 2/5 (d) 1/2

4. Fifteen coupons are numbered 1, 2, 3, …., 15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupon is 9, is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) none of these

5. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to getting 9 heads, then the probability of getting 2 heads is

(a) $15/2^8$ (b) $2/15$ (c) $15/2^{13}$ (d) $15/2^{17}$

6. A and B each throw a die. The probability that A's throw is not greater than B's is

(a) 1/6 (b) 5/12 (c) 1/2 (d) 7/12

7. The probability that an archer hits the target when it is windy is 2/5, when it is not windy his probability of hitting the target is 7/10. On any shot the probability of gust of wind is 3/10. The probability that there is no gust of wind when he missed the target, is equal to

(a) $\frac{5}{13}$ (b) $\frac{18}{39}$ (c) $\frac{7}{13}$ (d) $\frac{23}{39}$

8. If A and B are independent events such that $P(A) = 0.3$, $P(A \cup \bar{B}) = 0.8$, then $P(B)$ equals to

(a) 1/7 (b) 2/7 (c) 3/4 (d) 1/4

9. For three events A, B and C, P (exactly one of the events A or B occurs) = P (exactly one of the events B or C occurs) = P (exactly one of the events C or A occurs) = p and P (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events A, B and C occurring is

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{2}$

(c) $\frac{3p+p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

10. The probability that at least one of the events A and B happens is 3/5. Probability of their simultaneous happening is 1/5. Value of $P(\bar{A})+P(\bar{B})$ is equal to

(a) 2/5 (b) 4/5 (c) 6/5 (d) 7/5

11. One number is selected at random from first two hundred positive integers. The probability that it is divisible by 6 or 8, is equal to

(a) $\frac{29}{100}$ (b) $\frac{33}{100}$ (c) $\frac{43}{100}$ (d) $\frac{1}{4}$

12. Two fair and ordinary dice are rolled simultaneously. The probability of getting the sum of outcomes of the dice as a multiple of 4, is equal to

(a) 5/9 (b) 1/4 (c) 1/9 (d) 1/3

13. The probabilities of happening of the events A and B are 1/4 and 1/2 respectively. If the probability of happening of A and B simultaneously is $\frac{7}{50}$, then probability of neither A nor B happening is equal to

(a) $\frac{39}{100}$ (b) $\frac{1}{4}$ (c) $\frac{11}{100}$ (d) $\frac{17}{100}$

14. A five digit number (having all different digits) is formed using the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. The probability that the formed number either begins or ends with an odd digit, is equal to

(a) 5/6 (b) 1/6 (c) 1/3 (d) 2/3

15. The probabilities of solving a problem by students A, B and C independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. If they start solving the given problem independently, then the probability that at least two of them will solve the problem successfully, is equal to

(a) $\frac{5}{24}$ (b) $\frac{9}{24}$ (c) $\frac{7}{24}$ (d) $\frac{11}{24}$

16. Three numbers are chosen at random without replacement from the set of integers {1, 2, …, 10}. The probability that the minimum of the chosen numbers is 3 or the maximum of the chosen numbers is 7, is equal to

(a) $\frac{23}{120}$ (b) $\frac{13}{120}$ (c) $\frac{13}{60}$ (d) $\frac{11}{40}$

17. Two numbers are selected simultaneously from the set {6, 7, 8, 9, ……, 40}. If the sum of selected numbers is even then, the probability that both the selected numbers are odd, is equal to

(a) $\frac{11}{51}$ (b) $\frac{40}{51}$ (c) $\frac{51}{91}$ (d) $\frac{8}{17}$

18. A bag contains 4 red, 5 white and 6 black balls. Three balls are selected from this bag simultaneously. The probability that one of the colour will be missing in the selected balls, is equal to

(a) $\frac{301}{455}$ (b) $\frac{366}{455}$ (c) $\frac{261}{455}$ (d) $\frac{109}{455}$

19. In a certain city 40% of the people have brown hair, 25% have brown eyes and 15% have brown eyes as well as brown hair. If a person selected at random has brown hair, the probability that the person also has brown eyes, is equal to

(a) 2/5 (b) 1/4 (c) 1/2 (d) 3/8

20. In a bag there are 15 red and 5 white balls. Two balls are drawn in succession, without replacement. The first drawn ball is found to be red. The probability that second ball is also red, is equal to

(a) 3/10 (b) 7/10 (c) 5/19 (d) 14/19

21. A and B play a game where each is asked to select a number from 1 to 25. If the numbers selected by A and B are same, both of them win a prize. The probability that they win their third prize on 5^{th} game is equal to

(a) $\frac{6\cdot(24)^2}{(25)^5}$ (b) $\frac{6\cdot(21)^2}{(25)^5}$ (c) $\frac{(24)^2}{(25)^2}$ (d) $\frac{(21)^2}{(25)^5}$

22. A and B are two events such that $P(A) = 1/2$, $P(B) = 2/3$. Then which of the following is not correct?

(a) $P(A\cup B)\geq\frac{2}{3}$ (b) $P(A\cap B')\leq\frac{1}{3}$

(c) $\frac{1}{6}\leq P(A\cap B)\leq\frac{1}{2}$ (d) none of these

23. A bag contains 4 tickets numbered 1, 2, 3, 4 and another bag contains 6 tickets numbered 2, 4, 6, 7, 8, 9. One bag is chosen and a ticket is drawn. The probability that the ticket bears the number 4, is equal to

(a) $\frac{5}{12}$ (b) $\frac{5}{24}$ (c) $\frac{7}{12}$ (d) $\frac{19}{24}$

24. A bag contains four tickets marked with numbers 112, 121, 211, 222. One ticket is drawn at random from the bag. Let E_i $(i = 1, 2, 3)$ denote the event that i^{th} digit on the selected ticket is 2, then which of the following is not correct?

(a) E_1 and E_2 are independent.
(b) E_1 and E_3 are independent.
(c) E_3 and E_1 are independent.
(d) E_1, E_2 and E_3 are independent.

25. A real estate man has eight master keys to open several new homes. Only one master key will open any given home. If 40% of these homes are usually left unlocked, the probability that the real estate man can get into a specific home, if it is given that he selected 3 keys randomly before leaving his office, is equal to

(a) 5/8 (b) 3/8 (c) 3/4 (d) 1/4

26. Two persons are selected at random from n persons seated in a row $(n \geq 3)$. The probability that the selected persons are not seated consecutively, is equal to

(a) $\frac{n-2}{n}$ (b) $\frac{n-1}{n}$ (c) $\frac{n+2}{n+3}$ (d) $\frac{n-2}{n-1}$

27. Three numbers are selected simultaneously from the set {1, 2, 3, …., 25}. The probability that the product of selected numbers is divisible by 4, is equal to

(a) $\frac{1}{115}$ (b) $\frac{98}{115}$ (c) $\frac{773}{1150}$ (d) $\frac{963}{1150}$

28. Two squares are chosen from the squares of an ordinary chess board. It is given that the selected squares do not belong to the same row or column. The probability that the chosen squares are of same colour, is equal to

(a) $\frac{25}{49}$ (b) $\frac{32}{49}$ (c) $\frac{25}{64}$ (d) $\frac{1}{2}$

29. A committee consists of 9 experts taken from three institutions A, B and C, of which 2 are from A, 3 from B and 4 from C. If three experts resign from the committee, then the probability of exactly two of the resigned experts being from the same institution, is equal to

(a) $\frac{4}{7}$ (b) $\frac{25}{84}$ (c) $\frac{55}{84}$ (d) $\frac{37}{84}$

30. A bag B_1 has 3 white and 2 red balls. Another bag B_2 has 4 white and 6 red balls. A ball is drawn randomly from bag B_1 and without seeing it's colour, is being put in bag B_2. Now a ball is drawn from bag B_2. The probability of both the drawn balls, of being same colour, is

(a) $\frac{41}{55}$ (b) $\frac{31}{55}$ (c) $\frac{29}{55}$ (d) $\frac{33}{55}$

Multiple Correct Answer Type

31. The probability that a 50 year-old man will be alive at 60 is 0.83 and the probability that a 45 year-old woman will be alive at 55 is 0.87. Then

(a) the probability that both will be alive for the next 10 years is 0.7221.
(b) at least one of them will be alive for the next 10 years is 0.9779.
(c) at least one of them will be alive for the next 10 years is 0.8230.
(d) the probability that both will be alive for the next 10 years is 0.6320.

32. A random variable x takes values 0, 1, 2, 3, . . . with probability proportional to $(x+1)\left(\frac{1}{5}\right)^x$, then

(a) $P(x=0)=\frac{16}{25}$ (b) $P(x\leq 1)=\frac{112}{125}$

(c) $P(x\geq 1)=\frac{9}{25}$ (d) none of these

33. Cards are drawn one by one in a pack of well shuffled cards without replacement until all the cards are drawn, then

(a) chance of getting a spade at the 13^{th} trial is 1/13.
(b) chance of getting a spade at the 13^{th} trial is 1/4.
(c) chance of getting last card as a spade is 1/4.
(d) chance of getting king at the 5^{th} trial and queen at the 10^{th} trial is $\frac{4}{663}$.

34. A consignment of 15 record players contains 4 defective record players. The record players are selected at random, one by one and examined. The one examined are not put back. Then

(a) Probability of getting exactly 3 defective in the examination of 8 record players is $\frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}$.

(b) Probability that 9^{th} one examined is the last defective is $\frac{8}{195}$.

(c) Probability that 9^{th} examined record player is defective given that there were 3 defective in the first 8 players examined is 1/7.

(d) Probability that 9^{th} one examined is last defective is $\frac{8}{197}$.

35. Which of the following statements are true?

(a) The probability that birthday of twelve people will fall in twelve calendar months $= \frac{12!}{(12)^{12}}$.

(b) The probability that birthday of six people will fall in exactly two calendar months $= \frac{{}^{12}C_2(2^6-2)}{(12)^6}$.

(c) The probability that birthday of six people will fall in exactly two calendar months $= \frac{{}^{12}C_3(2^7-2)}{(12)^7}$.

(d) The probability that birthdays of n ($n \le 365$) people are different $= \frac{{}^{365}P_n}{(365)^n}$.

36. A five digit number with distinct digits is formed by using the digits 0, 1, 2, 3, 4, 5. The probability that the number is divisible by 3 is

(a) $\frac{{}^6P_5}{6^5}$ (b) $\frac{9}{25}$ (c) $\frac{5!+4\times4!}{5\times5!}$ (d) $\frac{{}^6C_5}{6^5}$

37. If E_1, E_2 are two events such that $P(E_1)=\frac{1}{4}$, $P(E_2/E_1) = 1/2$ and $P(E_1/E_2)=\frac{1}{4}$, then

(a) E_1 and E_2 are independent.
(b) E_1 and E_2 are exhaustive.
(c) E_2 is twice as likely to occur as E_1.
(d) Probabilities of the events $E_1 \cap E_2$, E_1, E_2 are in G.P.

38. Let an ordinary coin be tossed 15 times. If P_r denotes the probability of getting r tails. P_k is maximum, when $k =$

(a) 7 (b) 6 (c) 8 (d) 3

Matrix-Match Type

39. There are 10 pairs of shoes in a shoe rack from which 4 shoes are taken at random. If $P(E)$ denotes the probability of the event E, then match the following.

Column I		Column II	
A.	P(getting no pair)	p.	99/323
B.	P(getting at least one pair)	q.	96/323
C.	P(getting exactly two pairs)	r.	224/323
D.	P(getting exactly one pair)	s.	3/323

40. Five unbiased cubical dice are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dice, then match the probabilities given in the column II corresponding to the events given in the column I.

Column I		Column II	
A.	$m = 3$	p.	$\left(\frac{2}{3}\right)^5$
B.	$n = 4$	q.	$\left(\frac{2}{3}\right)^5+\left(\frac{1}{3}\right)^5-\left(\frac{1}{2}\right)^4$
C.	$2 \le m \le 4$	r.	$\left(\frac{5}{6}\right)^5-\left(\frac{1}{3}\right)^5$
D.	$m = 2$ and $n = 5$	s.	$\left(\frac{2}{3}\right)^5-\left(\frac{1}{2}\right)^5$

Numerical Value Type

41. Two numbers are selected at random from the set of first 100 natural numbers. The probability that the product obtained is divisible by 3 is k, then $\frac{150k}{83} =$

42. Functions are formed from set $A = \{1, 2, 3\}$ to set $B = \{1, 2, 3, 4, 5\}$ and one function is selected at random. If P be the probability that function satisfying $f(i) \le f(j)$ whenever $i < j$, then value of 25 P is equal to

43. If the sides of triangle are decided by throwing a die thrice, the probability that the triangle is isosceles or equilateral is $1/k$, then $k =$

44. Two squares are chosen at random on a chess board. If the chance that may have a contact at corner is $\frac{\mu}{144}$, then μ must be equal to

45. Three tangents are drawn at random to a given circle. The odds against the circle being inscribed in the triangle formed by them is $K:1$, then $K=$

SOLUTIONS

1. (a) : As we are considering all possible six digit numbers, half of these would have sum of their digits to be even and half of these would have sum of their digits to be odd. Hence, the required probability would be 1/2.

2. (c) : There are 64 small squares on a chessboard.

$\therefore$ Total number of ways to choose two squares

$= {}^{64}C_2 = 32 \cdot 63$

For favourable ways we must choose two consecutive small squares from any row or any column

$\therefore$ Number of favorable ways $= (7 \cdot 8)2$

So, required probability $= \frac{7 \cdot 8 \cdot 2}{32 \cdot 63} = \frac{1}{18}$

3. (d) : If n_1, n_2, n_3 and n_4 be the numbers on the drawn balls (in that order) then, clearly n_1 and n_4 are unequal. Thus, there are only two cases $n_1 < n_4$ or $n_1 > n_4$

Thus, required probability $= \frac{1}{2}$

4. (d) : Total ways $= 15^7$

For favourable ways, we must choose 7 coupons numbered from 1 to 9 so that '9' is selected at least once. Thus, total number of favourable ways are, $9^7 - 8^7$

So, required probability $= \frac{9^7 - 8^7}{15^7}$

5. (c) : Let coin was tossed n times and X be the random variable representing the number of heads appearing in n trials.

$\therefore \quad P(X = 7) = P(X = 9)$ [Given]

$\Rightarrow \quad {}^nC_7 (1/2)^7 (1/2)^{n-7} = {}^nC_9 (1/2)^{n-9} (1/2)^9$

$\Rightarrow \quad {}^nC_7 = {}^nC_9 \Rightarrow n = 16$

Now, $P(X=2) = {}^{16}C_2 (1/2)^2 (1/2)^{14} = \frac{{}^{16}C_2}{2^{16}} = \frac{16 \cdot 15}{2^{17}} = \frac{15}{2^{13}}$

6. (d) :

B's throw	*A*'s throw	Total numbers
1	2, 3, 4, 5, 6	5
2	3, 4, 5, 6	4
3	4, 5, 6	3
4	5, 6	2
5	6	1
		Total = 15

$\therefore \quad P(A > B) = \frac{15}{36}$ Hence, $P(A \le B) = 1 - \frac{15}{36} = \frac{7}{12}$

7. (c) : Let E_1 : There is a gust of wind.

E_2 : There is no gust of wind.

A : Archer misses the target.

$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$= \frac{3}{10} \cdot \frac{3}{5} + \frac{7}{10} \cdot \frac{3}{10} = \frac{39}{100}$

Now, required probability

$= P(E_2 / A)$

$= \frac{P(E_2 \cap A)}{P(A)} = \frac{P(E_2) \cdot P(A/E_2)}{P(A)} = \frac{\frac{7}{10} \cdot \frac{3}{10}}{\frac{39}{100}} = \frac{7}{13}$

8. (b) : $P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A)P(\bar{B})$

$\Rightarrow \quad 0.3 + (1 - x) - 0.3(1 - x) = 0.8$, where $P(B) = x$ (say)

$\Rightarrow \quad (1 - x)(0.7) = 0.5 \quad \Rightarrow 1 - x = \frac{5}{7} \Rightarrow x = 1 - \frac{5}{7} = \frac{2}{7}$

9. (a) : Given, $P(A) + P(B) - 2P(A \cap B) = p$ ... (i)

$P(B) + P(C) - 2P(B \cap C) = p$ ...(ii)

$P(C) + P(A) - 2P(A \cap C) = p$ ...(iii)

Adding (i), (ii) and (iii), we get

$[P(A) + P(B) + P(C) - (P(A \cap B) + P(B \cap C) + P(C \cap A))] = \frac{3p}{2}$ and $P(A \cap B \cap C) = p^2$

Hence, $P(A \cup B \cup C) = \frac{3p}{2} + p^2 = \frac{3p + 2p^2}{2}$

10. (c) : Given, $P(A \cup B) = \frac{3}{5}$, $P(A \cap B) = \frac{1}{5}$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 2 - P(A \cap B) - P(\bar{A}) - P(\bar{B})$

$\Rightarrow \quad P(\bar{A}) + P(\bar{B}) = 2 - P(A \cap B) - P(A \cup B)$

$= 2 - \frac{1}{5} - \frac{3}{5} = \frac{6}{5}$

11. (d) : Total number of integers that are divisible by 6 or 8 = 33 + 25 − 8 = 50

Thus, required probability $= \frac{50}{200} = \frac{1}{4}$

12. (b)

13. (a) : $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{7}{50}$

Now, $P(\bar{A} \cap \bar{B}) = P(\bar{A}) + P(\bar{B}) - P(\bar{A} \cup \bar{B})$

$= 1 - P(A) + 1 - P(B) - (1 - P(A \cap B))$

$= 1 - \frac{1}{4} - \frac{1}{2} + \frac{7}{50} = \frac{39}{100}$

14. (a) : Total formed numbers that begins with an odd digit $= {}^5C_1 \cdot {}^8P_4 = 5 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

Total formed numbers that ends with an odd digit
$= {}^5C_1 \cdot {}^8P_4 = 5 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

Total formed numbers that begins with an odd digit and also ends with an odd digit
$= {}^5C_2 \cdot 2! \cdot {}^7P_3 = 5 \cdot 4 \cdot 7 \cdot 6 \cdot 5$

Thus, total formed numbers that begins with an odd digit or ends with an odd digit is equal to $5 \cdot 7 \cdot 6 \cdot 60$

Total formed numbers $= {}^9P_5 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

Thus, required probability $= \dfrac{5}{6}$

15. (c)

16. (d) : Let A : Minimum of the chosen numbers is 3.
B : Maximum of the chosen numbers is 7.

$$\therefore \quad P(A) = \frac{{}^7C_2}{{}^{10}C_3} = \frac{21}{120}, \; P(B) = \frac{{}^6C_2}{{}^{10}C_3} = \frac{15}{120},$$

$$P(A \cap B) = \frac{{}^3C_1}{{}^{10}C_3} = \frac{3}{120}$$

Thus, required probability

$$= P(A) + P(B) - P(A \cap B) = \frac{11}{40}$$

17. (d) : Total number of even numbers in the set is 18 and total number of odd numbers is 17.
Let A : Sum of selected numbers is even.
B : Selected numbers are odd.

$$\therefore \quad P(A) = \frac{{}^{18}C_2 + {}^{17}C_2}{{}^{35}C_2}, \; P(A \cap B) = \frac{{}^{17}C_2}{{}^{35}C_2}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{{}^{17}C_2}{{}^{18}C_2 + {}^{17}C_2} = \frac{136}{289} = \frac{8}{17}$$

18. (a) : (i) Selected balls can be W W B or W B B (when red colour is missing).
Corresponding probability

$$= \frac{{}^5C_2 \cdot {}^6C_1 + {}^5C_1 \cdot {}^6C_2}{{}^{15}C_3} = \frac{135}{{}^{15}C_3}$$

(ii) Selected balls can be R R W or R W W (when black colour is missing).
Corresponding probability

$$= \frac{{}^4C_2 \cdot {}^5C_1 + {}^4C_1 \cdot {}^5C_2}{{}^{15}C_3} = \frac{70}{{}^{15}C_3}$$

(iii) Selected balls can be R B B or R R B (when white colour is missing).
Corresponding probability

$$= \frac{{}^4C_1 \cdot {}^6C_2 + {}^4C_2 \cdot {}^6C_1}{{}^{15}C_3} = \frac{96}{{}^{15}C_3}$$

Thus, required probability $= \dfrac{301}{455}$

19. (d) : Let E_1 : Person has brown hair.
E_2: Person has brown eyes.

Given, $P(E_1) = \dfrac{40}{100} = \dfrac{2}{5}$, $P(E_2) = \dfrac{25}{100} = \dfrac{1}{4}$,

$$P(E_1 \cap E_2) = \frac{15}{100} = \frac{3}{20}$$

$$\therefore \quad P(E_2 / E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{3/20}{2/5} = \frac{3}{8}$$

20. (d) : Total number of ways of selecting the second ball $= {}^{19}C_1 = 19$

Total number of ways of selecting the second ball suitably $= {}^{14}C_1 = 14$

Thus, required probability $= \dfrac{14}{19}$

21. (a) : Probability of winning the prize in single game

$$= \frac{25}{25^2} = \frac{1}{25}$$

In this case first 4 games, must result in exactly two prizes and 5^{th} game must result in prize.
Thus, required probability

$$= {}^4C_2 \left(\frac{1}{25}\right)^2 \cdot \left(\frac{24}{25}\right)^2 \cdot \frac{1}{25} = \frac{6 \cdot 24^2}{(25)^5}$$

22. (d) : $P(A \cup B) \geq \max \{P(A), P(B)\} = \dfrac{2}{3}$

$P(A \cap B') \leq \min \{P(A), P(B')\} = \dfrac{1}{3}$

$P(A \cap B) \geq \max \{0, (P(A) + P(B) - 1)\} = \dfrac{1}{6}$

and $P(A \cap B) \leq \min \{P(A), P(B)\} = \dfrac{1}{2}$

Thus, all the statements are correct.

23. (b) : Let E_1 : First bag is chosen, $P(E_1) = \dfrac{1}{2}$

E_2 : Second bag is chosen, $P(E_2) = \dfrac{1}{2}$

A : Drawn number is 4.
Now, $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$$= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{6} = \frac{5}{24}$$

24. (d) : $P(E_1) = \dfrac{2}{4} = \dfrac{1}{2}$, $P(E_2) = \dfrac{2}{4} = \dfrac{1}{2}$, $P(E_3) = \dfrac{2}{4} = \dfrac{1}{2}$

$$\therefore \quad P(E_1 \cap E_2) = \frac{1}{4} = P(E_1) \cdot P(E_2)$$

$\Rightarrow$ E_1 and E_2 are independent.

Now, $P(E_2 \cap E_3) = \dfrac{1}{4} = P(E_2) \cdot P(E_3)$

$\Rightarrow$ E_2 and E_3 are independent.

Again, $P(E_1 \cap E_3) = \frac{1}{4} = P(E_1) \cdot P(E_3)$

$\Rightarrow$ E_1 and E_3 are independent.

$$P(E_1 \cap E_2 \cap E_3) = \frac{1}{4} \neq P(E_1) \cdot P(E_2) \cdot P(E_3)$$

$\Rightarrow$ E_1, E_2 and E_3 are not independent.

25. (a) : Let E_1: Specific home is locked.

E_2 : Specific home is unlocked.

A : Real estate man get into the home.

$$P(E_2) = \frac{40}{100} = \frac{2}{5},\ P(E_1) = \frac{3}{5}$$

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$$

$$= \frac{3}{5} \cdot \frac{{}^7C_2}{{}^8C_3} + \frac{2}{5} \cdot 1 = \frac{25}{40} = \frac{5}{8}$$

26. (a) : Total ways of selecting two persons $= {}^nC_2$

$$= \frac{n(n-1)}{2}$$

Total ways of selecting two consecutively seated persons

$= {}^{n-1}C_1 = (n-1)$

Thus, required probability $= 1 - \frac{(n-1)2}{n(n-1)} = \frac{n-2}{n}$

27. (c) : Product can't be divisible by 4 in the following mutually exclusive cases;

(i) All the selected numbers are odd

Corresponding ways $= {}^{13}C_3 = 286$

(ii) Two of the selected numbers are odd and another is a multiple of 2 but not multiple of 4

Corresponding ways $= {}^{13}C_2 \cdot {}^6C_1 = 468$

Thus, required probability $= 1 - \frac{(286+468)}{{}^{25}C_3} = \frac{773}{1150}$

28. (a) : Let the first chosen square is white. In this case total number of squares that don't belong to the row or column associated with the selected square is 49. Out of these 25 are white and 24 are black.

Thus, corresponding probability $= \frac{1}{2} \cdot \frac{25}{49}$

Similarly, if the first square is black, then the corresponding probability $= \frac{1}{2} \cdot \frac{25}{49}$

Thus, required probability $= \frac{1}{2}\left(\frac{25}{49} + \frac{25}{49}\right) = \frac{25}{49}$

29. (c) : There are three mutually exclusive cases:

(i) 2 are from A and another is either from B or C.

Corresponding probability $= \frac{{}^2C_2 \cdot ({}^3C_1 + {}^4C_1)}{{}^9C_3} = \frac{7}{84}$

(ii) 2 are from B and another is either from A or C.

Corresponding probability $= \frac{{}^3C_2 \cdot ({}^2C_1 + {}^4C_1)}{{}^9C_3} = \frac{18}{84}$

(iii) 2 are from C and another is either from A or B.

Corresponding probability $= \frac{{}^4C_2 \cdot ({}^2C_1 + {}^3C_1)}{{}^9C_3} = \frac{30}{84}$

Thus, required probability $= \frac{7+18+30}{84} = \frac{55}{84}$

30. (c) : Let E_1 : Ball drawn from bag B_1 is white.

E_2 : Ball drawn from bag B_1 is red.

A : Both the drawn balls are of same colour.

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$$

$$= \frac{3}{5} \cdot \frac{5}{11} + \frac{2}{5} \cdot \frac{7}{11} = \frac{29}{55}$$

31. (a, b) : The probability that both will be alive for next 10 years = 0.83 × 0.87 = 0.7221

The probability that at least one of them will be alive is

1– P(none of them remains alive for 10 years)

= 1 – (1 – 0.83)(1 – 0.87) = 1 – (0.17 × 0.13)

= 0.9779

32. (a, c) : We have, $P(X = x) \propto (x+1)\left(\frac{1}{5}\right)^x$

since, $\sum_{x=0}^{\infty} P(X = x) = 1$

$$\Rightarrow k\left\{1 + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + 4\left(\frac{1}{5}\right)^3 + \ldots.\infty\right\} = 1 \Rightarrow k = \frac{16}{25}$$

(a) $P(X = 0) = k(0+1)\left(\frac{1}{5}\right)^0 = k = \frac{16}{25}$

(b) $P(X \leq 1) = P(X = 0) + P(X = 1) = \frac{26}{25}$

(c) $P(X \geq 1) = 1 - P(X = 0) = 1 - k = \frac{9}{25}$

33. (b,c,d) : Let us make a sample space of 52 dimensions *i.e.*, each point of sample space is an ordered point of the form $\{x_1, x_2, \ldots., x_{52}\}$.Total number of sample points are 52!. Number of sample points on which x_{13} is spade is ${}^{13}C_1 \times 51!$

$\therefore$ Probability of getting spade on $x_{13} = \frac{{}^{13}C_1 \times 51!}{52!} = \frac{1}{4}$

Similarly number of sample points on x_{52} is spade

$= {}^{13}C_1 \times 51!$

$\therefore$ Probability of getting spade on last trial = 1/4

Number of sample points having king at x_5 and queen x_{10} will be ${}^4C_1 \times {}^4C_1 \times 50!$

Required Probability $= \frac{16 \times 50!}{52!} = \frac{4}{663}$

34. (a, b, c) : Let A be the event of getting exactly 3 defective in the examination of 8 record players and B be the event 9^{th} record player is defective.

$\therefore \quad P(A \cap B) = P(A)P(B/A)$

$$P(A) = \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}, P\left(\frac{B}{A}\right) = \frac{1}{7}$$

Probability of 9^{th} one examined is the last defective

$$= \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} \times \frac{1}{7} = \frac{8}{195}$$

35. (a, b, d) : (a) Required probability

$$= \frac{12}{12} \times \frac{11}{12} \times \frac{10}{12} \times .... \times \frac{1}{12} = \frac{12!}{(12)^{12}}$$

(b) Two months can be selected in ${}^{12}C_2$ ways. For each selection every person has two choices in 2^6 ways but it includes two cases in which all persons were born in the same month.

Total number of favourable cases = ${}^{12}C_2(2^6 - 2)$

$$\text{Required probability} = \frac{{}^{12}C_2(2^6 - 2)}{(12)^6}$$

(d) Required probability

$$= \frac{365}{365} \times \frac{364}{365} \times ..... \times \frac{365-(n-1)}{365} = \frac{{}^{365}P_n}{(365)^n}$$

36. (b, c) : Let S be the sample space .Then $n(S) = 5 \times {}^5P_4$. If 0 is present, then the number of 5 digit numbers divisible by 3 is $4 \times 4! = 96$. If 0 is absent, then the number of 5 digit numbers divisible by 3 is 5! = 120.

$\therefore \quad$ Required probability $= \frac{216}{600} = \frac{9}{25}$

37. (a, c, d) : $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} \Rightarrow P(E_1 \cap E_2) = \frac{1}{8}$

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} \Rightarrow P(E_2) = \frac{1}{2}$$

38. (a, c) : $P_k = \frac{{}^{15}C_k}{2^{15}}$

It is maximum, when $k = \frac{15-1}{2}$ or $\frac{15+1}{2}$

39. $\mathbf{A \to r, B \to p, C \to s, D \to q}$

(A) P(getting no pair) $= \frac{20}{20} \cdot \frac{18}{19} \cdot \frac{16}{18} \cdot \frac{14}{17} = \frac{224}{323}$

(B) P(getting at least one pair) $= 1 - \frac{224}{323} = \frac{99}{323}$

(C) P(getting exactly two pairs) $= \frac{{}^{10}C_2}{{}^{20}C_4} = \frac{3}{323}$

(D) P(getting exactly one pair) $= 1 - \left[\frac{224}{323} + \frac{3}{323}\right] = \frac{96}{323}$

40. $\mathbf{A \to s, B \to s, C \to r, D \to q}$

(A) When $m = 3$, the number appearing on upper face of any die can be 3, 4, 5 or 6 *i.e.*, maximum 4 cases.

$$P(m = 3) = P(m \geq 3) - P(m \geq 4) = \frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(B) When $n = 4$, the number appearing on upper face of any die can be 1, 2, 3 or 4 *i.e.*, maximum 4 cases.

$$P(n = 4) = P(n \leq 4) - P(n \leq 3) = \frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(C) $P(2 \leq m \leq 4) = P(m \geq 2) - P(m \geq 5)$

$$= \frac{5^5 - 2^5}{6^5} = \left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$$

(D) $P(m = 2, n = 5) = P(2, 3, 4 \text{ or } 5) - P(2, 3 \text{ or } 4)$
$- P(3, 4 \text{ or } 5) + P(3 \text{ or } 4)$

$$= \frac{4^5 - 2 \times 3^5 + 2^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$$

41. (1) : Required probability $= \frac{{}^{33}C_2 + {}^{33}C_1 {}^{67}C_1}{{}^{100}C_2} = \frac{83}{150}$

42. (7) : Total number of functions = $5^3 = 125$

Number of functions satisfying $f(i) \leq f(j)$ if $i < j$

$= {}^5C_3 + {}^5C_2 (1 + 1) + {}^5C_1 = 35$

$\therefore \quad$ Required probability $= \frac{35}{125} = \frac{7}{25}$

43. (8) : Let the sides be a, b, c.

$a = b = 1, c = 1$
$a = b = 2, c = 1, 2, 3$
$a = b = 3, c = 1, 2, 3, 4, 5$
$a = b = 4, c = 1, 2, 3, 4, 5, 6$
$a = b = 5, c = 1, 2, 3, 4, 5, 6$
$a = b = 6, c = 1, 2, 3, 4, 5, 6$

The number of these triangles is $1 + 3 + 5 + 3 \times 6 = 27$

$\therefore \quad$ Required probability $= \frac{27}{6^3} = \frac{1}{8}$

44. (7) : Total cases of choosing two squares on a chess board = 64×63

Favourable cases = 4×1 (corner) + $24 \times 2 + 36 \times 4$
$= 4 \times 49$

$\therefore \quad$ Required probability $= \frac{4 \times 49}{64 \times 63} = \frac{7}{144} \Rightarrow \mu = 7$

45. (2) : $P(E) = \frac{2}{6} = \frac{1}{3}, \; P(\bar{E}) = \frac{2}{3}$

$\therefore \quad$ odds against = 2 : 1

WBJEE SOLVED PAPER 2020

CATEGORY-I (Q. 1 to Q. 50)

Carry 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ¼ mark will be deducted.

1. The equation of the plane through the point $(2,-1,-3)$ and parallel to the lines $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-4}$ and $\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2}$ is

(a) $8x + 14y + 13z + 37 = 0$
(b) $8x - 14y - 13z - 37 = 0$
(c) $8x - 14y - 13z + 37 = 0$
(d) $8x - 14y + 13z + 37 = 0$

2. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{2\sqrt{3}}{5}$ (b) $\frac{\sqrt{2}}{10}$ (c) $\frac{4}{5\sqrt{2}}$ (d) $\frac{\sqrt{5}}{6}$

3. Let $f(x) = \sin x + \cos ax$ be periodic function. Then

(a) 'a' is any real number.
(b) 'a' is any irrational number.
(c) 'a' is rational number.
(d) $a = 0$

4. The domain of $f(x)=\sqrt{\left(\frac{1}{\sqrt{x}}-\sqrt{(x+1)}\right)}$ is

(a) $x > -1$ (b) $(-1, \infty)\backslash\{0\}$
(c) $\left(0, \frac{\sqrt{5}-1}{2}\right]$ (d) $\left[\frac{1-\sqrt{5}}{2}, 0\right)$

5. Let $y = f(x) = 2x^2 - 3x + 2$. The differential of y when x changes from 2 to 1.99 is

(a) 0.01 (b) 0.18 (c) −0.05 (d) 0.07

6. If $\lim_{x\to 0}\left(\frac{1+cx}{1-cx}\right)^{1/x} = 4$, then $\lim_{x\to 0}\left(\frac{1+2cx}{1-2cx}\right)^{1/x}$ is

(a) 2 (b) 4 (c) 16 (d) 64

7. Let $f : R \to R$ be twice continuously differentiable or f'' exists and is continuous) such that $f(0) = f(1) = f'(0) = 0$. Then

(a) $f''(c) = 0$ for some $c \in R$
(b) there is no point for which $f''(x) = 0$
(c) at all points $f''(x) > 0$
(d) at all points $f''(x) < 0$

8. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 12$. Then

(a) $f(x)$ has 13 non-zero real roots.
(b) $f(x)$ has exactly one real root.
(c) $f(x)$ has exactly one pair of imaginary roots.
(d) $f(x)$ has no real root.

9. Let $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$. Then

(a) $x^2y_2 + xy_1 + n^2y = 0$
(b) $xy_2 - xy_1 + 2n^2y = 0$
(c) $x^2y_2 + 3xy_1 - n^2y = 0$
(d) $xy_2 + 5xy_1 - 3y = 0$

$\left(\text{Here } y_2 \equiv \frac{d^2y}{dx^2},\ y_1 \equiv \frac{dy}{dx}\right)$

10. Let $\phi(x) = f(x) + f(1 - x)$ and $f''(x) < 0$ in $[0, 1]$, then

(a) ϕ is monotonic increasing in $\left[0, \frac{1}{2}\right]$ and monotonic decreasing in $\left[\frac{1}{2}, 1\right]$
(b) ϕ is monotonic increasing in $\left[\frac{1}{2}, 1\right]$ and monotonic decreasing in $\left[0, \frac{1}{2}\right]$
(c) ϕ is neither increasing nor decreasing in any sub interval of $[0, 1]$
(d) ϕ is increasing in $[0, 1]$

11. $\int \frac{f(x)\phi'(x)+\phi(x)f'(x)}{(f(x)\phi(x)+1)\sqrt{f(x)\phi(x)-1}}\,dx =$

(a) $\sin^{-1}\sqrt{\dfrac{f(x)}{\phi(x)}}+c$

(b) $\cos^{-1}\sqrt{(f(x))^2-(\phi(x))^2}+c$

(c) $\sqrt{2}\tan^{-1}\sqrt{\dfrac{f(x)\,\phi(x)-1}{2}}+c$

(d) $\sqrt{2}\tan^{-1}\sqrt{\dfrac{f(x)\,\phi(x)+1}{2}}+c$

Where c is the constant of integration.

12. The value of $\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}x\,dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx$ is equal to

(a) 27 (b) 54 (c) −54 (d) 0

13. $\int_0^2[x^2]\,dx$ is equal to

(a) 1 (b) $5-\sqrt{2}-\sqrt{3}$

(c) $3-\sqrt{2}$ (d) 8/3

14. If the tangent to the curve $y^2=x^3$ at (m^2, m^3) is also a normal to the curve at (M^2, M^3), then the value of mM is

(a) $-\frac{1}{9}$ (b) $-\frac{2}{9}$ (c) $-\frac{1}{3}$ (d) $-\frac{4}{9}$

15. If $x^2+y^2=a^2$, then $\int_0^a\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx=$

(a) $2\pi a$ (b) πa (c) $\frac{1}{2}\pi a$ (d) $\frac{1}{4}\pi a$

16. Let f, be a continuous function in [0, 1], then $\lim_{n\to\infty}\sum_{j=0}^{n}\frac{1}{n}f\left(\frac{j}{n}\right)$ is

(a) $\frac{1}{2}\int_0^{\frac{1}{2}}f(x)\,dx$ (b) $\int_{\frac{1}{2}}^{1}f(x)\,dx$

(c) $\int_0^1 f(x)\,dx$ (d) $\int_0^{\frac{1}{2}}f(x)\,dx$

17. Let f be a differentiable function with $\lim_{x\to\infty}f(x)=0$. If $y'+yf'(x)-f(x)f'(x)=0$, $\lim_{x\to\infty}y(x)=0$, then (where $y'\equiv\dfrac{dy}{dx}$)

(a) $y+1=e^{f(x)}+f(x)$ (b) $y-1=e^{f(x)}+f(x)$

(c) $y+1=e^{-f(x)}+f(x)$ (d) $y-1=e^{-f(x)}+f(x)$

18. If $x\sin\left(\frac{y}{x}\right)dy=\left[y\sin\left(\frac{y}{x}\right)-x\right]dx$, $x>0$ and $y(1)=\dfrac{\pi}{2}$ then the value of $\cos\left(\frac{y}{x}\right)$ is

(a) 1 (b) $\log x$ (c) e (d) 0

19. Let $f(x)=1-\sqrt{(x)^2}$ where the square root is to be taken positive, then

(a) f has no extrema at $x=0$

(b) f has minima at $x=0$

(c) f has maxima at $x=0$

(d) f' exists at 0

20. If the function $f(x)=2x^3-9ax^2+12a^2x+1[a>0]$ attains its maximum and minimum at p and q respectively such that $p^2=q$, then a is equal to

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 3

21. If a and b are arbitrary positive real numbers, then the least possible value of $\dfrac{6a}{5b}+\dfrac{10b}{3a}$ is

(a) 4 (b) $\frac{6}{5}$ (c) $\frac{10}{3}$ (d) $\frac{68}{15}$

22. If $2\log(x+1)-\log(x^2-1)=\log 2$, then $x=$

(a) only 3 (b) −1 and 3

(c) only −1 (d) 1 and 3

23. The number of complex numbers p such that $|p|=1$ and imaginary part of p^4 is 0, is

(a) 4 (b) 2

(c) 8 (d) infinitely many

24. The equation $z\bar{z}+(2-3i)z+(2+3i)\bar{z}+4=0$ represents a circle of radius

(a) 2 units (b) 3 units (c) 4 units (d) 6 units

25. The expression ax^2+bx+c (a, b and c are real) has the same sign as that of a for all x if

(a) $b^2-4ac>0$ (b) $b^2-4ac\neq 0$

(c) $b^2-4ac\leq 0$

(d) b and c have the same sign as that of a

26. In a 12 storied building, 3 persons enter a lift cabin. It is known that they will leave the lift at different floors. In how many ways can they do so if the lift does not stop at the second floor?

(a) 36 (b) 120 (c) 240 (d) 720

27. If the total number of m-element subsets of the set $A=\{a_1, a_2, \ldots, a_n\}$ is k times the number of m element subsets containing a_4, then n is

(a) $(m-1)k$ (b) mk

(c) $(m+1)k$ (d) $(m+2)k$

28. Let $I(n) = n^n$, $J(n) = 1 \cdot 3 \cdot 5 \ldots\ldots (2n - 1)$ for all $(n > 1)$, $n \in N$, then

(a) $I(n) > J(n)$ (b) $I(n) < J(n)$

(c) $I(n) = J(n)$ (d) $I(n) = \frac{1}{2} J(n)$

29. If $c_0, c_1, c_2, \ldots\ldots, c_{15}$ are the Binomial coefficients in the expansion of $(1 + x)^{15}$, then the value of $\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \ldots\ldots\ldots + 15\frac{c_{15}}{c_{14}}$ is

(a) 1240 (b) 120 (c) 124 (d) 140

30. Let $A = \begin{pmatrix} 3-t & 1 & 0 \\ -1 & 3-t & 1 \\ 0 & -1 & 0 \end{pmatrix}$ and $\det A = 5$, then

(a) $t = 1$ (b) $t = 2$

(c) $t = -1$ (d) $t = -2$

31. Let $A = \begin{pmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{pmatrix}$. The value of x for which the matrix A is not invertible is

(a) 6 (b) 12 (c) 3 (d) 2

32. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 real matrix with $\det A = 1$. If the equation $\det (A - \lambda I_2) = 0$ has imaginary roots (I_2 be the identity matrix of order 2), then

(a) $(a + d)^2 < 4$ (b) $(a + d)^2 = 4$

(c) $(a + d)^2 > 4$ (d) $(a + d)^2 = 16$

33. If $\begin{vmatrix} a^2 & bc & c^2 + ac \\ a^2 + ab & b^2 & ca \\ ab & b^2 + bc & c^2 \end{vmatrix} = ka^2b^2c^2$, then $k =$

(a) 2 (b) -2 (c) -4 (d) 4

34. If $f: S \to R$ where S is the set of all non-singular matrices of order 2 over R and $f\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = ad - bc$, then

(a) f is bijective mapping.

(b) f is one-one but not onto.

(c) f is onto but not one-one.

(d) f is neither one-one nor onto.

35. Let the relation ρ be defined on R by $a \rho b$ holds if and only if $a - b$ is zero or irrational, then

(a) ρ is equivalence relation.

(b) ρ is reflexive and symmetric but is not transitive.

(c) ρ is reflexive and transitive but is not symmetric.

(d) ρ is reflexive only.

36. The unit vector in ZOX plane, making angles 45° and 60° respectively with $\vec{\alpha} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{\beta} = \hat{j} - \hat{k}$ is

(a) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ (b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k}$

(c) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$ (d) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

37. Four persons A, B, C and D throw an unbiased die, turn by turn, in succession till one gets an even number and win the game. What is the probability that A wins if A begins?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{7}{12}$ (d) $\frac{8}{15}$

38. A rifleman is firing at a distant target and has only 10% chance of hitting it. The least number of rounds he must fire to have more than 50% chance of hitting it at least once, is

(a) 5 (b) 7 (c) 9 (d) 11

39. $\cos 2x + 7 = a(2 - \sin x)$ can have a real solution for

(a) all real values of a (b) $a \in [2, 6]$

(c) $a \in (-\infty, 2) \backslash \{0\}$ (d) $a \in (0, \infty)$

40. The differential equation of the family of curves $y = e^x(A \cos x + B \sin x)$ where A, B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - 9x = 13$ (b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(c) $\frac{d^2y}{dx^2} + 3y = 4$ (d) $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - xy = 0$

41. The equation $r\cos\left(\theta - \frac{\pi}{3}\right) = 2$ represents

(a) a circle (b) a parabola

(c) an ellipse (d) a straight line

42. The locus of the centre of the circles which touch both the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4ax$ externally is

(a) a circle (b) a parabola

(c) an ellipse (d) a hyperbola

43. Let each of the equations $x^2 + 2xy + ay^2 = 0$ and $ax^2 + 2xy + y^2 = 0$ represent two straight lines passing through the origin. If they have a common line, then the other two lines are given by

(a) $x - y = 0, x - 3y = 0$

(b) $x + 3y = 0, 3x + y = 0$

(c) $3x + y = 0, 3x - y = 0$

(d) $3x - 2y = 0, x + y = 0$

44. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at P and Q respectively. The point O divides the segment PQ in the ratio

(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

45. Area in the first quadrant between the ellipses $x^2 + 2y^2 = a^2$ and $2x^2 + y^2 = a^2$ is

(a) $\frac{a^2}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$ (b) $\frac{3a^2}{4}\tan^{-1}\frac{1}{2}$

(c) $\frac{5a^2}{2}\sin^{-1}\frac{1}{2}$ (d) $\frac{9\pi a^2}{2}$

46. The equation of circle of radius $\sqrt{17}$ units, with centre on the positive side of x-axis and through the point (0, 1) is

(a) $x^2 + y^2 - 8x - 1 = 0$

(b) $x^2 + y^2 + 8x - 1 = 0$

(c) $x^2 + y^2 - 9y + 1 = 0$

(d) $2x^2 + 2y^2 - 3x + 2y = 4$

47. The length of the chord of the parabola $y^2 = 4ax$ $(a > 0)$ which passes through the vertex and makes an acute angle α with the axis of the parabola is

(a) $\pm 4a\cot\alpha\,\text{cosec}\alpha$ (b) $4a\cot\alpha\,\text{cosec}\alpha$

(c) $-4a\cot\alpha\,\text{cosec}\alpha$ (d) $4a\,\text{cosec}^2\alpha$

48. A double ordinate PQ of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is such that ΔOPQ is equilateral, O being the centre of the hyperbola. Then the eccentricity e satisfies the relation

(a) $1<e<\frac{2}{\sqrt{3}}$ (b) $e=\frac{2}{\sqrt{3}}$

(c) $e=\frac{\sqrt{3}}{2}$ (d) $e>\frac{2}{\sqrt{3}}$

49. If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, then the area of the rhombus $SBS'B'$ will be

(a) 12 sq. units (b) 48 sq. units

(c) 24 sq. units (d) 36 sq. units

50. The equation of the latus rectum of a parabola is $x + y = 8$ and the equation of the tangent at the vertex is $x + y = 12$. Then the length of the latus rectum is

(a) $4\sqrt{2}$ units (b) $2\sqrt{2}$ units

(c) 8 units (d) $8\sqrt{2}$ units

CATEGORY-II (Q : 51 TO 65)

Carry 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

51. Let $y=\frac{1}{1+x+\ln x}$, then

(a) $x\frac{dy}{dx}+y=x$ (b) $x\frac{dy}{dx}=y(y\ln x-1)$

(c) $x^2\frac{dy}{dx}=y^2+1-x^2$ (d) $x\left(\frac{dy}{dx}\right)^2=y-x$

52. Consider the curve $y = be^{-x/a}$ where a and b are non-zero real numbers. Then

(a) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve at (0, 0).

(b) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve where the curve crosses the axis of y.

(c) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve at $(a, 0)$.

(d) $\frac{x}{a}+\frac{y}{b}=1$ is tangent to the curve at $(2a, 0)$.

53. The area of the region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ is

(a) $\frac{\pi^2}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{4}-\frac{1}{2}$ (d) $\frac{\pi^2}{3}$

54. In open interval $\left(0,\frac{\pi}{2}\right)$,

(a) $\cos x + x\sin x < 1$ (b) $\cos x + x\sin x > 1$

(c) no specific order relation can be ascertained between $\cos x + x\sin x$ and 1

(d) $\cos x+x\sin x<\frac{1}{2}$

55. If the line $y = x$ is a tangent to the parabola $y = ax^2 + bx + c$ at the point (1, 1) and the curve passes through (−1, 0), then

(a) $a = b = -1, c = 3$ (b) $a=b=\frac{1}{2}, c=0$

(c) $a=c=\frac{1}{4}, b=\frac{1}{2}$ (d) $a=0, b=c=\frac{1}{2}$

56. If the vectors $\vec{\alpha}=\hat{i}+a\hat{j}+a^2\hat{k}, \vec{\beta}=\hat{i}+b\hat{j}+b^2\hat{k}$ and $\vec{\gamma}=\hat{i}+c\hat{j}+c^2\hat{k}$ are three non-coplanar vectors and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$, then the value of abc is

(a) 1 (b) 0 (c) −1 (d) 2

57. Let z_1 and z_2 be two imaginary roots of $z^2 + pz + q = 0$, where p and q are real. The points z_1, z_2 and origin form an equilateral triangle if

(a) $p^2 > 3q$ (b) $p^2 < 3q$

(c) $p^2 = 3q$ (d) $p^2 = q$

58. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, where $ac \neq 0$ [a, b, c, d are all real], then $P(x)\cdot Q(x) = 0$ has

(a) at least two real roots (b) two real roots

(c) four real roots (d) no real root

59. Let $A = \{x \in R : -1 \leq x \leq 1\}$ and $f : A \to A$ be a mapping defined by $f(x) = x|x|$. Then f is

(a) injective but not surjective

(b) surjective but not injective

(c) neither injective nor surjective

(d) bijective

60. Let $f(x) = \sqrt{x^2 - 3x + 2}$ and $g(x) = \sqrt{x}$ be two given functions. If S be the domain of fog and T be the domain of gof, then

(a) $S = T$ (b) $S \cap T = \phi$

(c) $S \cap T$ is a singleton.

(d) $S \cap T$ is an interval.

61. Let ρ_1 and ρ_2 be two equivalence relations defined on a non-void set S. Then

(a) both $\rho_1 \cap \rho_2$ and $\rho_1 \cup \rho_2$ are equivalence relations.

(b) $\rho_1 \cap \rho_2$ is equivalence relation but $\rho_1 \cup \rho_2$ is not so.

(c) $\rho_1 \cup \rho_2$ is equivalence relation but $\rho_1 \cap \rho_2$ is not so.

(d) neither $\rho_1 \cap \rho_2$ nor $\rho_1 \cup \rho_2$ is equivalence relation.

62. Consider the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The portion of the tangent at any point of the curve intercepted between the point of contact and the directrix subtends at the corresponding focus an angle of

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

63. A line cuts the x-axis at $A(7, 0)$ and the y-axis at $B(0, -5)$. A variable line PQ is drawn perpendicular to AB cutting the x-axis at $P(a, 0)$ and the y-axis at $Q(0, b)$. If AQ and BP intersect at R, the locus of R is

(a) $x^2 + y^2 + 7x + 5y = 0$

(b) $x^2 + y^2 + 7x - 5y = 0$

(c) $x^2 + y^2 - 7x + 5y = 0$

(d) $x^2 + y^2 - 7x - 5y = 0$

64. Let $0 < \alpha < \beta < 1$. Then $\lim_{n\to\infty} \sum_{k=1}^{n} \int_{1/(k+\beta)}^{1/(k+\alpha)} \frac{dx}{1+x}$ is

(a) $\log_e \frac{\beta}{\alpha}$ (b) $\log_e \frac{1+\beta}{1+\alpha}$

(c) $\log_e \frac{1+\alpha}{1+\beta}$ (d) ∞

65. $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{(x-1)} \right)$

(a) Does not exist (b) is 1

(c) is 1/2 (d) is 0

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Samurai Sudoku (February)

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CATEGORY-III (Q : 66 TO 75)

Carry 2 marks each and one or more option(s) is/are correct. If all correct answers are not marked and no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers. If any wrong option is marked or if any combination including a wrong option is marked, the answer will considered wrong, but there is no negative marking for the same and zero mark will be awarded.

66. Let $f(x)=\frac{1}{3}x\sin x-(1-\cos x)$. The smallest positive integer k such that $\lim_{x\to 0}\frac{f(x)}{x^k}\neq 0$ is

(a) 4 (b) 3 (c) 2 (d) 1

67. Tangent is drawn at any point $P(x, y)$ on a curve, which passes through (1, 1). The tangent cuts X-axis and Y-axis at A and B respectively. If $AP : BP = 3 : 1$, then

(a) the differential equation of the curve is $3x\frac{dy}{dx}+y=0$

(b) the differential equation of the curve is $3x\frac{dy}{dx}-y=0$

(c) the curve passes through $\left(\frac{1}{8}, 2\right)$

(d) the normal at (1, 1) is $x + 3y = 4$

68. The area of the figure bounded by the parabola $x = -2y^2, x = 1 - 3y^2$ is

(a) $\frac{1}{3}$ square unit (b) $\frac{4}{3}$ square units

(c) 1 square unit (d) 2 square units

69. A particle is projected vertically upwards. If it has to stay above the ground for 12 seconds, then

(a) velocity of projection is 192 ft/sec

(b) greatest height attained is 600 ft

(c) velocity of projection is 196 ft/sec

(d) greatest height attained is 576 ft

70. The equation $x^{(\log_3 x)^2-\frac{9}{2}\log_3 x+5}=3\sqrt{3}$ has

(a) at least one real root

(b) exactly one real root

(c) exactly one irrational root

(d) complex roots

71. In a certain test, there are n questions. In this test 2^{n-i} students gave wrong answers to at least i questions, where $i = 1, 2, ..., n$. If the total number of wrong answers given is 2047, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

72. A and B are independent events. The probability that both A and B occur is 1/20 and the probability that neither of them occurs is 3/5. The probability of occurrence of A is

(a) 1/2 (b) 1/10 (c) 1/4 (d) 1/5

73. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is

(a) $\frac{x}{2}-\frac{y}{3}=1$ (b) $\frac{x}{-2}+\frac{y}{1}=1$

(c) $-\frac{x}{3}+\frac{y}{2}=1$ (d) $\frac{x}{1}-\frac{y}{2}=1$

74. Consider a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at any point. The locus of the midpoint of the portion intercepted between the axes is

(a) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (b) $\frac{x^2}{4}+\frac{y^2}{2}=1$

(c) $\frac{1}{3x^2}+\frac{1}{4y^2}=1$ (d) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

75. Let $y=\frac{x^2}{(x+1)^2(x+2)}$. Then $\frac{d^2y}{dx^2}$ is

(a) $2\left[\frac{3}{(x+1)^4}-\frac{3}{(x+1)^3}+\frac{4}{(x+2)^3}\right]$

(b) $3\left[\frac{2}{(x+1)^3}+\frac{4}{(x+1)^2}-\frac{5}{(x+2)^3}\right]$

(c) $\frac{6}{(x+1)^3}-\frac{4}{(x+1)^2}+\frac{3}{(x+1)^3}$

(d) $\frac{7}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{2}{(x+1)^3}$

ANSWER KEYS

1.	(None)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(c)
6.	(c)	**7.**	(a)	**8.**	(b)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(d)	**13.**	(b)	**14.**	(d)	**15.**	(c)
16.	(c)	**17.**	(c)	**18.**	(b)	**19.**	(c)	**20.**	(a)
21.	(a)	**22.**	(a)	**23.**	(c)	**24.**	(b)	**25.**	(c)
26.	(d)	**27.**	(b)	**28.**	(a)	**29.**	(b)	**30.**	(d)
31.	(c)	**32.**	(a)	**33.**	(d)	**34.**	(d)	**35.**	(b)
36.	(b)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(b)
41.	(d)	**42.**	(d)	**43.**	(b)	**44.**	(b)	**45.**	(a)
46.	(a)	**47.**	(b)	**48.**	(d)	**49.**	(c)	**50.**	(d)
51.	(b)	**52.**	(b)	**53.**	(c)	**54.**	(b)	**55.**	(c)
56.	(c)	**57.**	(c)	**58.**	(a)	**59.**	(d)	**60.**	(d)
61.	(b)	**62.**	(c)	**63.**	(c)	**64.**	(b)	**65.**	(c)
66.	(c)	**67.**	(a,c)	**68.**	(b)	**69.**	(a,d)	**70.**	(a,c)
71.	(b)	**72.**	(c,d)	**73.**	(a,b)	**74.**	(d)	**75.**	(a)

(For detailed solutions, refer to MTG WB JEE Chapterwise Explorer)

JEE Main

NUMERICAL VALUE TYPE QUESTIONS

Exam on
5th, 7th to 9th
&
11th April, 2020

1. If $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$, then number of values of (α, β) which satisfy both the equations is/are _____.

2. The value of term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is _____.

3. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number _____.

4. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals _____.

5. The value of $\lim_{x\to 0}\left((\sin x)^{1/x} + (1/x)^{\sin x}\right)$, where $x > 0$, is _____.

6. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is _____.

7. The real number x when added to its inverse gives the minimum value of the sum at x equals to _____.

8. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is _____.

9. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is _____.

10. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is _____.

11. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is _____.

12. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is _____.

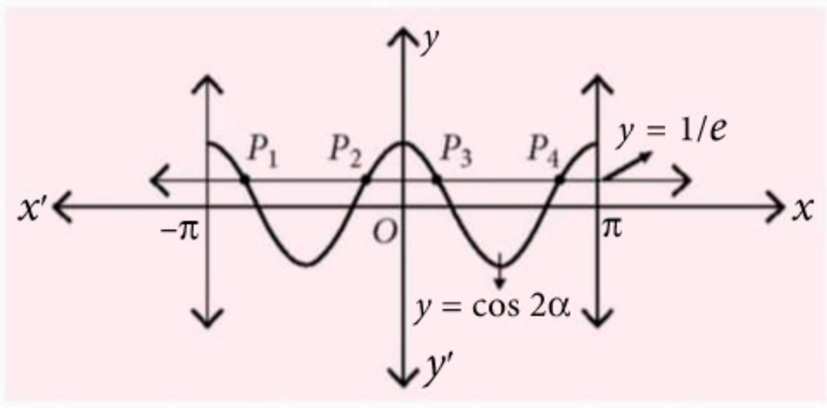
13. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is _____.

14. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is _____.

15. If $x + y = k$ is normal to $y^2 = 12x$, then k is _____.

SOLUTIONS

1. (4) : Given that $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$



Now, $\cos(\alpha - \beta) = 1 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta$

$\therefore$ $\cos(\alpha + \beta) = 1/e \Rightarrow \cos 2\alpha = 1/e$

and $2\alpha \in [-2\pi, 2\pi]$

$\Rightarrow$ There will be two values of α satisfying $\cos 2\alpha = 1/e$ in $[0, \pi]$ and two in $[-\pi, 0]$

$\Rightarrow$ There will be four values of α in $[-\pi, \pi]$ and correspondingly four values of β.

Hence, there are four sets of (α, β).

2. (210) : $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$

$= \left\{\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)}\right\}^{10}$

$= (x^{1/3} - x^{-1/2})^{10}$ $\therefore$ $T_{r+1} = (-1)^r\, {}^{10}C_r\, x^{\frac{20-5r}{6}}$

Thus $\frac{20-5r}{6} = 0 \Rightarrow r = 4$ $\therefore$ Term $= {}^{10}C_4 = 210$.

3. (601) : We have letters A, C, H, I, N, S

No. of words starting with A = 5!

No. of words starting with C = 5!

No. of words starting with H = 5!

No. of words starting with I = 5!

No. of words starting with N = 5!

Total words = 5! + 5! + 5! + 5! + 5! = 5(5 !) = 600

So, required rank of SACHIN = 600 + 1 = 601

4. (6) : Let A (1, 1) ; B (2, 4) and $P(x_1, y_1)$ divides line segment AB in the ratio 3 : 2.

$$\therefore \quad x_1 = \frac{3(2)+2(1)}{5} = \frac{8}{5}, \quad y_1 = \frac{3(4)+2(1)}{5} = \frac{14}{5}$$

$2x + y = k$ passes through $P(x_1, y_1)$.

$$\therefore\ 2\times\frac{8}{5}+\frac{14}{5}=k \ \Rightarrow\ k=6$$

5. (1) : We have, $\lim\limits_{x\to 0}\left[(\sin x)^{1/x}+(1/x)^{\sin x}\right]$

$$=\lim_{x\to 0}(\sin x)^{1/x}+\lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}$$

$$=0+e^{\lim\limits_{x\to 0}\sin x\log\left(\frac{1}{x}\right)} \qquad [\because\ |\sin x|<1 \text{ when } x\to 0]$$

$$=e^{\lim\limits_{x\to 0}\frac{-\log x}{\operatorname{cosec} x}}=e^{\lim\limits_{x\to 0}\frac{-1/x}{-\operatorname{cosec} x\cot x}} \text{ [Using L' Hospital's rule]}$$

$$=e^{\lim\limits_{x\to 0}\frac{\sin x}{x}\cdot\tan x}=e^0=1$$

6. (2) : We have, $f(x) = x^3 + e^{x/2}$ and $g(f(x)) = x$

On differentiating, we get $g'(f(x))\, f'(x) = 1$

Putting $x = 0$, we get $g'(f(0))\, f'(0) = 1$

But $f(0) = 1$ and $f'(0)=\frac{1}{2}$ $\left(\because\ f'(x)=3x^2+\frac{1}{2}e^{x/2}\right)$

$$\therefore \quad g'(1)=\frac{1}{f'(0)}=\frac{1}{1/2}=2$$

7. (1) : $f(x) = x + 1/x$

$f'(x) = 1 - 1/x^2$ and $f''(x) = \frac{2}{x^3}$

Now $f'(x) = 0$

$\Rightarrow \quad x = \pm 1 \quad \therefore \quad f''(1) > 0$

$\Rightarrow \quad x = 1$ is point of minima.

8. (2.5) : Let $\mathrm{I}=\int_{e^{-1}}^{e^2}\left|\frac{\log_e x}{x}\right|dx$

We know that for $\frac{1}{e}<x<1$, $\log_e x < 0$ and hence $\frac{\log_e x}{x}<0$ and for $1 < x < e^2$, $\log_e x > 0$ and hence $\frac{\log_e x}{x}>0$

$$\therefore \quad I=\int_{1/e}^{1}-\frac{\log_e x}{x}dx+\int_1^{e^2}\frac{\log_e x}{x}dx$$

$$=-\frac{1}{2}\left[(\log_e x)^2\right]_{1/e}^{1}+\frac{1}{2}\left[(\log_e x)^2\right]_1^{e^2}$$

$$=-\frac{1}{2}\left[0-(-1)^2\right]+\frac{1}{2}\left[(2)^2-0\right]=\frac{1}{2}+2=\frac{5}{2}=2.5$$

9. (3) : We have, $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$

$\Rightarrow \quad 3\{|a|^2+|b|^2+|c|^2\}-(\vec{a}+\vec{b}+\vec{c})^2=9$

$\Rightarrow \quad 3\times 3-(\vec{a}+\vec{b}+\vec{c})^2=9$

$\Rightarrow \quad \vec{a}+\vec{b}+\vec{c}=0 \ \Rightarrow\ \vec{b}+\vec{c}=-\vec{a}$

$\therefore \quad |2\vec{a}+5\vec{b}+5\vec{c}|=|2\vec{a}+5(-\vec{a})|=3|\vec{a}|=3$

10. (7) : The given plane contains the given line if the normal to the plane is perpendicular to the line.

$\because$ 2(1) – 4(1) + 1(2) = 0 (satisfied)

and the point (4, 2, k) on the line lies in the plane if 2(4) – 4(2) + k = 7 $\Rightarrow$ k = 7

11. (5) : Line $5x - 2y + 6 = 0$ meets y-axis at (0,3).

$\Rightarrow \quad Q \equiv (0, 3)$

The length of tangent from Q (0, 3) to the circle $x^2 + y^2 + 6x + 6y - 2 = 0$ is $\sqrt{0+9+0+18-2}=\sqrt{25}=5$

12. (0.4) : No. of horses = 5

$\therefore$ Probability that Mr. A does not select the winning horse $=\frac{4}{5}\times\frac{3}{4}$

Probability that Mr. A select the winning horse

$$=1-P(\bar{A})=1-\frac{12}{20}=\frac{2}{5}=0.4$$

13. (4) : Let $|x| = t$. Then $t^2 - 3t + 2 = 0$

$\Rightarrow \quad (t - 1)\,(t - 2) = 0 \ \Rightarrow\ t = 1, 2$

If $t = 1$, $|x| = 1$ $\therefore\ x = \pm 1$

If $t = 2$, $|x| = 2$ $\therefore\ x = \pm 2$

$\therefore$ The given equation has 4 solutions.

14. (8) : Using A.M. - G.M. inequality, we have

$$\frac{a^{-5}+a^{-4}+3a^{-3}+1+a^8+a^{10}}{8}\geq(1)^{1/8}$$

$\Rightarrow \quad a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10} \geq 8$

Hence, the least value is 8 and it realized also at $a = 1$.

15. (9) : $y = mx + k$ is normal to $y^2 = 12x$, then $k = -2am - am^3$

Here, $a = 3$ and $x + y = k$

$\Rightarrow \quad y = (-1)\,x + k \ \Rightarrow\ m = -1$

$\therefore \quad k = -2(3)\,(-1) - 3(-1)^3 = 9$

Monthly Test Drive-10 CLASS XII ANSWER KEY

1. (d)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (b)
6. (d)	**7.** (b,c,d)	**8.** (b)	**9.** (b)	**10.** (a,c)
11. (a,b,c)	**12.** (b,c,d)	**13.** (a,b,c)	**14.** (b)	**15.** (c)
16. (c)	**17.** (891)	**18.** (9)	**19.** (0.8)	**20.** (0.4)

FOCUS JEE MAIN 2020 PRACTICE PAPER

with Numerical Value Type Questions

1. If $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$ for $x\neq 0, y\neq 0$ then D is

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

2. The sum of the series
$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - ..... + {}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}{}^{20}C_{10}$

3. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$. Then,

(a) $f^{-1}(y)=\frac{y-3}{4}$ (b) $f^{-1}(y)=\frac{3y+4}{3}$
(c) $f^{-1}(y)=4+\frac{y+3}{4}$ (d) f^{-1} does not exist

4. The function $f(x)=\frac{\ln(\pi+x)}{\ln(e+x)}$ is

(a) increasing on $(0, \infty)$
(b) decreasing on $(0, \infty)$
(c) increasing on $(0, \pi/e)$, decreasing on $(\pi/e, \infty)$
(d) decreasing on $(0, \pi/e)$, increasing on $(\pi/e, \infty)$

5. If $\frac{\omega-\bar{\omega}z}{1-z}$ is purely real, where $\omega = \alpha + i\beta$, $\beta \neq 0$ and $z \neq 1$, then the set of the values of z is

(a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
(c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$

6. Let $I=\int\frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J=\int\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$.

Then, for an arbitrary constant C, the value of $J - I$ equals

(a) $\frac{1}{2}\log\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right|+C$

(b) $\frac{1}{2}\log\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right|+C$

(c) $\frac{1}{2}\log\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right|+C$

(d) $\frac{1}{2}\log\left|\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right|+C$

7. The no. of real solutions of

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

(a) zero (b) one (c) two (d) infinite

8. The equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ has

(a) no root (b) one root
(c) two equal roots (d) infinitely many roots

9. Let $y(x)$ be the solution of the differential equation $(x \ln x)\frac{dy}{dx}+y=2x\ln x$, $(x \geq 1)$. Then $y(e)$ is equal to

(a) 2 (b) $2e$ (c) e (d) 0

10. Let $a_1, a_2, a_3, ...$ be terms of an A.P. If $\frac{a_1+a_2+...+a_p}{a_1+a_2+...+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals

(a) 41/11 (b) 7/2 (c) 2/7 (d) 11/41

11. Events A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then set of possible values of x is

(a) $\left[\frac{1}{3},\frac{2}{3}\right]$ (b) $\left[\frac{1}{3},\frac{13}{3}\right]$

(c) $[0, 1]$ (d) $\left[\frac{1}{3},\frac{1}{2}\right]$

12. $\lim\limits_{x\to 1}\dfrac{\sqrt{1-\cos 2(x-1)}}{x-1}$

(a) exists and it equals $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exists because $x-1\to 0$

(d) does not exist because the left hand limit is not equal to the right hand limit.

13. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect a point x on the circumference of the circle, then $2r$ equals

(a) $\sqrt{PQ\cdot RS}$ (b) $\dfrac{PQ+RS}{2}$

(c) $\dfrac{2PQ+RS}{PQ+RS}$ (d) $\sqrt{\dfrac{PQ^2+RS^2}{2}}$

14. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio $5 : 10 : 14$. Then $n =$

(a) 4 (b) 5 (c) 6 (d) 7

15. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point $B(1, 3, 4)$ in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining $A(3, 1, 6)$ and $B(1, 3, 4)$.

(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

16. If $f(x)=x^n$, then the value of

$f(1)-\dfrac{f'(1)}{1!}+\dfrac{f''(1)}{2!}-\dfrac{f'''(1)}{3!}+....+\dfrac{(-1)^n f^n(1)}{n!}$ is

(a) 2^{n-1} (b) 0 (c) 1 (d) 2^n

17. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1$, is

(a) 27/2 (b) 27 (c) 27/4 (d) 18

18. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane

(a) $x - y - 2z = 1$ (b) $x - 2y - z = 1$

(c) $x - y - z = 1$ (d) $2x - y - z = 1$

19. Let $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors and $\vec{p},\vec{q},\vec{r}$ are vectors defined by the relations

$\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{r}=\dfrac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$

then the value of the expression

$(\vec{a}+\vec{b})\cdot\vec{p}+(\vec{b}+\vec{c})\cdot\vec{q}+(\vec{c}+\vec{a})\cdot\vec{r}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

20. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".

Statement-1 : r is equivalent to either q or p.

Statement-2 : r is equivalent to $\sim(p\leftrightarrow\sim q)$.

(a) Statement-1 is true, Statement-2 is false.

(b) Statement-1 is false, Statement-2 is true.

(c) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

Numerical Value Type

21. If $A=\begin{bmatrix}-4 & -1\\ 3 & 1\end{bmatrix}$, then the determinant of the matrix $(A^{2020} - 2A^{2019} - A^{2018})$ is

22. Let the straight line $x = b$ divide the area enclosed by $y = (1 - x)^2$, $y = 0$ and $x = 0$ into two parts R_1 $(0\le x\le b)$ and $R_2(b\le x\le 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equals to

23. The number of values of θ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ such that $\theta\neq\dfrac{n\pi}{5}$ for $n = 0, \pm1, \pm2$ and $\tan\theta = \cot5\theta$ as well as $\sin2\theta = \cos4\theta$ is

24. Let S_k, $k = 1, 2, ..., 100$, denote the sum of the infinite geometric series whose first term is $\dfrac{k-1}{k!}$ and the common ratio is $1/k$. Then the value of $\dfrac{100^2}{100!}+\sum\limits_{k=1}^{100}|(k^2-3k+1)S_k|$ is

25. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

SOLUTIONS

1. (d): $D=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+x & 1\\ 1 & 1 & 1+y\end{vmatrix}$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$=\begin{vmatrix}1 & 0 & 0\\ 1 & x & 0\\ 1 & 0 & y\end{vmatrix}=1(xy-0)=xy$$

Hence, D is divisible by both x and y.

2. (d): $\because \quad {}^{20}C_0+{}^{20}C_1x+.....+{}^{20}C_{10}x^{10}+.....$

$$.....+{}^{20}C_{20}x^{20}=(1+x)^{20}$$

After putting $x = -1$, we get

$${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....$$

$$.....+{}^{20}C_{10}-{}^{20}C_{11}-{}^{20}C_{12}+.....+{}^{20}C_{20}=0$$

$$\Rightarrow 2({}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-{}^{20}C_9)+{}^{20}C_{10}=0$$

$$\Rightarrow {}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-{}^{20}C_9+{}^{20}C_{10}=\frac{1}{2}{}^{20}C_{10}$$

3. (a): Let $f(x_1)=f(x_2)$, $x_1, x_2 \in N$

$\Rightarrow \quad 4x_1+3=4x_2+3 \quad \Rightarrow \quad x_1=x_2$

Thus, $f(x_1)=f(x_2) \quad \Rightarrow \quad x_1=x_2$. Hence the function is one-one. Let $y \in Y$ be any number. Then, $y = 4k + 3$, for some $k \in N$. Thus $y = f(k)$, where $k \in N$.

So, corresponding to any $y \in Y$ we have $x \in N$. Hence, the function is onto and the function, being both one-one and onto is invertible.

$$y=4x+3 \quad \Rightarrow \quad x=\frac{y-3}{4} \quad \therefore \quad f^{-1}(y)=\frac{y-3}{4}$$

4. (b): We have, $f(x)=\dfrac{\ln(\pi+x)}{\ln(e+x)}$

$$\therefore \quad f'(x)=\frac{\left(\frac{1}{\pi+x}\right)\ln(e+x)-\frac{1}{(e+x)}\ln(\pi+x)}{[\ln(e+x)]^2}$$

$$=\frac{(e+x)\ln(e+x)-(\pi+x)\ln(\pi+x)}{(e+x)(\pi+x)(\ln(e+x))^2}<0 \text{ on } (0,\infty)$$

Since $1 < e < \pi$

$\therefore \quad f(x)$ decreases on $(0, \infty)$

5. (d): $\because \quad \dfrac{\omega-\bar{\omega}z}{1-z}$ is purely real

$$\therefore \quad \overline{\left(\frac{\omega-\bar{\omega}z}{1-z}\right)}=\left(\frac{\omega-\bar{\omega}z}{1-z}\right) \Rightarrow \frac{\bar{\omega}-\omega\bar{z}}{1-\bar{z}}=\frac{\omega-\bar{\omega}z}{1-z}$$

$$\Rightarrow \quad \bar{\omega}-\bar{\omega}z-\omega\bar{z}+\omega z\,\bar{z}=\omega-\omega\bar{z}-\bar{\omega}z+\bar{\omega}z\,\bar{z}$$

$$\Rightarrow \quad (\omega-\bar{\omega})(1-z\bar{z})=0 \Rightarrow 1-|z|^2=0$$

$\Rightarrow \quad |z|^2 = 1 \qquad (\because\ \omega = \alpha + i\beta \text{ and } \beta \neq 0)$

$\therefore \quad$ The required set is $\{z : |z| = 1, z \neq 1\}$.

6. (c): Set up the difference

$$J-I=\int\left(\frac{e^{-x}}{e^{-4x}+e^{-2x}+1}-\frac{e^{x}}{e^{4x}+e^{2x}+1}\right)dx$$

$$=\int\left(\frac{e^{3x}}{e^{4x}+e^{2x}+1}-\frac{e^{x}}{e^{4x}+e^{2x}+1}\right)dx=\int\frac{e^x(e^{2x}-1)}{e^{4x}+e^{2x}+1}dx$$

Putting $e^x = t \Rightarrow dx = dt/t$, we get

$$J-I=\int\frac{t^2-1}{t^4+t^2+1}dt=\int\frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-1}=\frac{1}{2}\log\left|\frac{t+\frac{1}{t}-1}{t+\frac{1}{t}+1}\right|+C$$

$$\therefore \quad J-I=\frac{1}{2}\log\left|\frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right|+C$$

7. (c): We must have, $x(x + 1) \geq 0$ ...(i)

and $0 \leq x^2 + x + 1 \leq 1$ ...(ii)

From (i), $x^2 + x + 1 \geq 1$

or $\quad 1 \leq x^2 + x + 1 \leq 1$ [From (ii)]

$\therefore \quad x^2 + x + 1 = 1$

$\Rightarrow \quad x(x + 1) = 0 \quad \therefore \quad x = 0 \text{ or } -1$

The given equation is true for these two values. Hence, the number of real solutions is two.

8. (a): $x-\dfrac{2}{x-1}=1-\dfrac{2}{x-1}$ ...(i)

For the validity of (i) $x \neq 1$.

Cancelling $-\dfrac{2}{x-1}$ from both sides, we get, $x = 1$ a contradiction

$\therefore \quad$ The given equation has no roots.

9. (a): The equation can be written as

$\dfrac{dy}{dx}+\left(\dfrac{1}{x\ln x}\right)y=2$, which is linear differential equation in y. Thus, I.F. $=e^{\int\frac{dx}{x\ln x}}=e^{\ln(\ln x)}=\ln x$

The solution is $y(\ln x)=\int 2(\ln x)+\lambda$

$= 2x\,(\ln x - 1) + \lambda$

At $x = 1$, we have $\lambda = 2$

The solution becomes $y \ln x = 2x(\ln x - 1) + 2$

Set $x = e$ in the above to obtain

$y = 2e\,(\ln e - 1) + 2 = 2$

The value of y at $x = e$, *i.e.* $y(e) = 2$

10. (d) : Given $a_1, a_2, a_3, \ldots$ be terms of A.P.

$$\frac{a_1+a_2+....+a_p}{a_1+a_2+...+a_q}=\frac{p^2}{q^2}\Rightarrow\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$$

$$\Rightarrow \quad \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

$\Rightarrow \quad [2a_1 + (p - 1)d]q = p[2a_1 + (q - 1)d]$

$\Rightarrow \quad 2a_1(q - p) = d[(q - 1)p - (p - 1)q]$

$\Rightarrow \quad 2a_1(q - p) = d(q - p) \Rightarrow 2a_1 = d$

$$\Rightarrow \quad \frac{a_6}{a_{21}}=\frac{a_1+5d}{a_1+20d}=\frac{a_1+10a_1}{a_1+40a_1}\Rightarrow\frac{a_6}{a_{21}}=\frac{11}{41}$$

11. (d) : A, B, C are mutually exclusive

$\therefore \quad 0 \le P(A) + P(B) + P(C) \le 1$ ...(i)

$0 \le P(A), P(B), P(C) \le 1$ ...(ii)

Now on solving (i) and (ii), we get $1/3 \le x \le 1/2$

12. (d) : $1 - \cos(2(x-1)) = 2\sin^2(x-1)$

or $\sqrt{1-\cos(2(x-1))}=\sqrt{2}\,|\sin(x-1)|$

$$\lim_{x\to1^-}\frac{\sqrt{1-\cos(2(x-1))}}{x-1}=\lim_{x\to1^-}\frac{\sqrt{2}\,|\sin(x-1)|}{x-1}$$

$$=\lim_{h\to0}\frac{\sqrt{2}\,|\sin(1-h-1)|}{1-h-1}\,(h>0)$$

$$=\sqrt{2}\lim_{h\to0}\frac{|\sin(-h)|}{-h}=-\sqrt{2}\lim_{h\to0}\frac{\sin h}{h}=-\sqrt{2}(1)=-\sqrt{2}$$

and $\lim_{x\to1^+}\frac{\sqrt{1-\cos(2(x-1))}}{x-1}=\lim_{x\to1^+}\frac{\sqrt{2}\,|\sin(x-1)|}{x-1}$

$$=\lim_{h\to0}\frac{\sqrt{2}\,|\sin(1+h-1)|}{1+h-1}\,(h>0)=\sqrt{2}\lim_{h\to0}\frac{|\sin h|}{h}$$

$$=\sqrt{2}\lim_{h\to0}\frac{\sin h}{h}=\sqrt{2}(1)=\sqrt{2}$$

$\therefore$ Left hand limit $\neq$ Right hand limit.

13. (a) : As the triangle PQR is a right angled triangle,

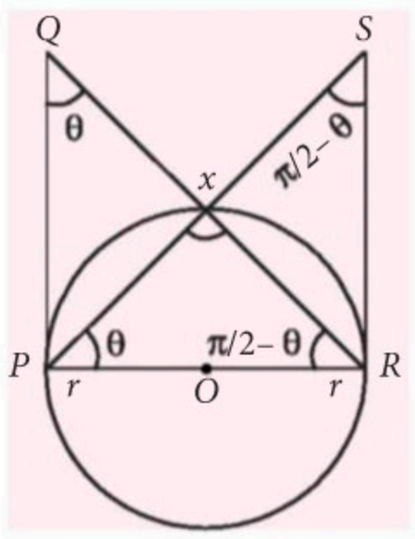
therefore $\frac{PQ}{PR}=\cot\theta$

$$\Rightarrow \quad \frac{PQ}{2r}=\cot\theta \qquad ...(i)$$

Also, as ΔPRS is a right angled triangle, therefore

$$\frac{RS}{PR}=\cot\left(\frac{\pi}{2}-\theta\right)$$

$$\Rightarrow \quad \frac{RS}{2r}=\tan\theta \qquad ...(ii)$$



From (i) and (ii), we get

$$\left(\frac{PQ}{2r}\right)\left(\frac{RS}{2r}\right)=\cot\theta\cdot\tan\theta=1$$

$(PQ)(RS) = (2r)^2 \Rightarrow 2r=\sqrt{(PQ)\cdot(R\ \ )}$

14. (c) : We have,

$^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} :: 5 : 10 : 14$

$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}}$ gives $\frac{(n+5)-r+1}{r}=2$

Also, $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r}$ gives $\frac{(n+5)-(r+1)+1}{r+1}=\frac{7}{5}$

Thus, $\frac{n+6}{r}=3$ and $\frac{n+6}{r+1}=\frac{12}{5}$

Solving for r, we obtain $r = 4$

Thus, $n + 6 = 12 \Rightarrow n = 6$

15. (b) : Let the image be (a, b, c)

Then by image formula, we have

$$\frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=-2\left(\frac{1-3+4-5}{3}\right)$$

$$\Rightarrow \quad \frac{a-1}{1}=\frac{b-3}{-1}=\frac{c-4}{1}=2$$

$\therefore \quad (a, b, c) = (3, 1, 6)$

Again the midpoint of $A(3, 1, 6)$ and $B(1, 3, 4)$ is $(2, 2, 5)$ & the equation of the plane is $x - y + z = 5$. As the point lies on the plane, so the plane bisects the segment AB. But it does not explain Statement-1.

16. (b) : We have, $f(x) = x^n$, so $f(1) = 1 = {}^nC_0$

Also, $f'(x) = nx^{n-1}$, so $-f'(1) = -n = -{}^nC_1$

$f''(x) = n(n - 1)x^{n-2}$, so $\frac{f''(1)}{2!}=\frac{n(n-1)}{2!}={}^nC_2$

$f^n(x) = n(n - 1) \ldots 1$, so $\frac{f^n(1)(-1)^n}{n!}=(-1)^n\,{}^nC_n$

$$\therefore \quad f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots+\frac{(-1)^n f^n(1)}{n!}$$

$= {}^nC_0 - {}^nC_1 + {}^nC_2 \ldots + (-1)^n C_n$

Now, $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

Putting $x = -1$ in both sides of (i), we get

$0 = C_0 - C_1 + C_2 - C_3 + \ldots$

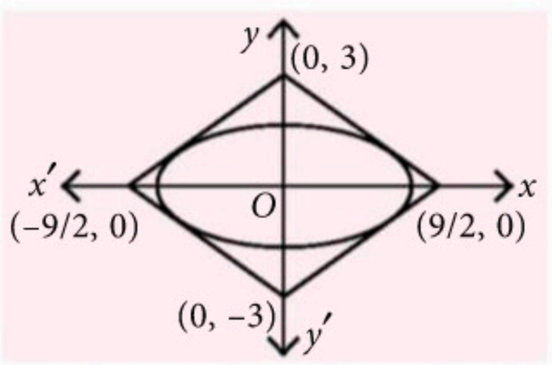
17. (b) : The ellipse is as described.

$$\frac{x^2}{9}+\frac{y^2}{5}=1$$

gives $a=3, b=\sqrt{5}$

Using $b^2 = a^2(1 - e^2)$, we have

$$1-e^2=\frac{5}{9} \Rightarrow e^2=\frac{4}{9}$$



$\Rightarrow \quad e = 2/3$

Equation of tangent at $(2, 5/3)$ is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1 \ \ i.e., \ \ \frac{x\cdot2}{9}+\frac{y}{5}\cdot\left(\frac{5}{3}\right)=1 \therefore \ \ \frac{2x}{9}+\frac{y}{3}=1$$

The points of intersection of the above line with axes are (0, 3) and (9/2, 0).
Using symmetry, picture can be completed.

$\therefore$ Required area $= \frac{1}{2}(3)(9/2)\times 4 = 27$

18. (d) : Equation of the plane of intersection of two spheres $S_1 = 0 = S_2$ is given by $S_1 - S_2 = 0$

$\Rightarrow \quad 10x - 5y - 5z = 5 \Rightarrow 2x - y - z = 1$

19. (d) : Given expression $= \sum(\vec{a}+\vec{b})\cdot\frac{(\vec{b}\times\vec{c})}{[\vec{a}\,\vec{b}\,\vec{c}]}$

$$= \sum\left(\frac{[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{b}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]}\right) = \sum(1+0) = 3$$

20. (a) : The given statement $r \equiv \sim p \leftrightarrow q$

The Statement-1 is $r \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

The Statement-2 is $r \equiv \sim (p \leftrightarrow \sim q)$

$\equiv (p \wedge q) \vee (\sim q \wedge \sim p)$

We can establish that Statement-1 is true but Statement-2 is false.

21. (–25) : We have,

$$A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$$

Also, $|A| = -1$

Now, $A^{2020} - 2A^{2019} - A^{2018} = A^{2018}(A^2 - 2A - I)$

$\therefore \quad |A^{2020} - 2A^{2019} - A^{2018}| = |A^{2018}|\,|A^2 - 2A - I|$

$$= |A|^{2018}\begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$$

22. (0.5) : $R_1 = \int_0^b (1-x)^2\,dx = \left[-\frac{(1-x)^3}{3}\right]_0^b = \frac{(b-1)^3+1}{3}$

$$R_2 = \int_b^1 (x-1)^2\,dx = \left[\frac{(x-1)^3}{3}\right]_b^1 = -\frac{(b-1)^3}{3}$$

We have, $R_1 - R_2 = \frac{1}{4}$

$$\Rightarrow \frac{(b-1)^3+1}{3} + \frac{(b-1)^3}{3} = \frac{1}{4} \Rightarrow \frac{2(b-1)^3}{3} = \frac{1}{4} - \frac{1}{3}$$

$$\Rightarrow \frac{2(b-1)^3}{3} = \frac{3-4}{12} = -\frac{1}{12} \Rightarrow (b-1)^3 = -\frac{1}{8}$$

$$\Rightarrow (b-1) = -\frac{1}{2} \quad \Rightarrow \quad b = \frac{1}{2} = 0.5$$

23. (3)

24. (3) : $S_k = \frac{k-1}{\lfloor k\left(1-\frac{1}{k}\right)} = \frac{1}{\lfloor k-1}, k \geq 2$

For $k = 1$, $S_1 = 0$

$$\therefore \quad \sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right| = 0 + \sum_{k=2}^{100}\left|(k^2-3k+1)\frac{1}{\lfloor k-1}\right|$$

$$= \sum_{k=2}^{100}\left|\frac{k-1}{\lfloor k-2} - \frac{k}{\lfloor k-1}\right| = 1 + \sum_{k=3}^{100}\left|\frac{k-1}{\lfloor k-2} - \frac{k}{\lfloor k-1}\right|$$

$$= 1 + \left(\frac{2}{\lfloor 1} - \frac{3}{\lfloor 2}\right) + \left(\frac{3}{\lfloor 2} - \frac{4}{\lfloor 3}\right) + \ldots + \left(\frac{99}{\lfloor 98} - \frac{100}{\lfloor 99}\right) = 1 + 2 - \frac{100}{\lfloor 99}$$

Thus, $\frac{100^2}{\lfloor 100} + \sum_{k=1}^{100}|(k^2-3k+1)S_k| = 3$

25. (5.5) : $\sigma_1^2 = 4, n_1 = 5, \bar{x}_1 = 2$

$\sigma_2^2 = 5, \; n_2 = 5, \; \bar{x}_2 = 4$

$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = \frac{5\times 2 + 5\times 4}{10} = 3$$

$d_1 = (\bar{x}_1 - \bar{x}_{12}) = -1, \quad d_2 = (\bar{x}_2 - \bar{x}_{12}) = 1$

$$\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1+n_2}}$$

$$= \sqrt{\frac{5\times 4 + 5\times 5 + 5\times 1 + 5\times 1}{10}} = \sqrt{\frac{55}{10}} = \sqrt{\frac{11}{2}}$$

$\therefore \quad \sigma^2 = \frac{11}{2} = 5.5$

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.
Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

80×		3	5–		2÷
	11+		1–		
9×	2	3–		30×	
		11+		2÷	
6	8×		13+		8+
10×				1	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

MONTHLY TEST DRIVE

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 10 : Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. A man is known to speak truth in 75% cases. If he throws an unbiased die and tells his friend that it is a six, then the probability that it is actually a six, is
(a) 1/6 (b) 1/8 (c) 3/4 (d) 3/8

2. If A and B are two events such that $P(A)=\frac{2}{5}$ and $P(A\cap B)=\frac{3}{20}$, then $P(A/\bar{A}\cup\bar{B})$ is equal to
(a) $\frac{8}{17}$ (b) $\frac{1}{4}$ (c) $\frac{5}{17}$ (d) $\frac{11}{20}$

3. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is
(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

4. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that the minimum of the two numbers is less than 4 is
(a) $\frac{1}{15}$ (b) $\frac{14}{15}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

5. There are n urns each containing $(n + 1)$ balls such that the i^{th} urn contains 'i' white balls and $(n + 1 - i)$ red balls. Let U_i be the event of selecting i^{th} urn, $i = 1, 2, 3, ..., n$ and W denotes the event of getting a white ball. If $P(U_i) \propto i$, where $i = 1, 2, 3,...., n$, then $\lim_{n\to\infty} P(W)$ is equal to
(a) 1 (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

6. A random variable X takes values $-1, 0, 1, 2$ with probabilities $\frac{1+3p}{4}, \frac{1-p}{4}, \frac{1+2p}{4}, \frac{1-4p}{4}$ respectively, where p varies over R. Then the minimum and maximum values of the mean of X are respectively
(a) $-\frac{7}{4}$ and $\frac{1}{2}$ (b) $-\frac{1}{16}$ and $\frac{5}{16}$
(c) $-\frac{7}{4}$ and $\frac{5}{16}$ (d) $-\frac{1}{16}$ and $\frac{5}{4}$

One or More Than One Option(s) Correct Type

7. If A and B are independent events such that $0 < P(A) < 1, 0 < P(B) < 1$, then
(a) A, B are mutually exclusive
(b) A and $\bar{B}$ are independent
(c) $\bar{A}, \bar{B}$ are independent
(d) $P(A/B) + P(\bar{A}/B) = 1$

8. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed, is
(a) 1/3 (b) 1/6 (c) 1/2 (d) 1/4

9. If X follows a binomial distribution with parameters $n = 8$ and $p = 1/2$, then $P(|X - 4| \le 2)$ equals
(a) $\frac{118}{128}$ (b) $\frac{119}{128}$ (c) $\frac{117}{128}$ (d) $\frac{113}{128}$

10. Let X and Y be two events such that $P(X/Y)=\frac{1}{2}, P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$.
Which of the following is(are) correct?

(a) $P(X \cup Y) = \frac{2}{3}$ (b) $P(\bar{X} \cap Y) = \frac{1}{3}$

(c) X and Y are independent

(d) X and Y are not independent

11. A random variable X follows binomial distribution with mean a and variance b. Then

(a) $a > b > 0$ (b) $\frac{a}{b} > 1$

(c) $\frac{a^2}{a-b}$ is an integer (d) $\frac{a^2}{a+b}$ is an integer

12. Two events A and B are such that the probability that atleast one of them occurs is 5/6 and both of them occurring simultaneously is 1/3. If the probability of not occurrence of B is 1/2, then

(a) A and B are equally likely

(b) A and B are independent

(c) $P(A/B)=2/3$

(d) $3P(A) = 4P(B)$

13. If A and B are two independent events such that $P(A) = 1/2$, $P(B) = 1/5$, then

(a) $P(A/B) = 1/2$ (b) $P\left(\frac{A}{A \cup B}\right) = \frac{5}{6}$

(c) $P\left(\frac{A \cap B}{\bar{A} \cup \bar{B}}\right) = 0$ (d) none of these

Comprehension Type

If a particular experiment be given n (a finite) independent trials. If the probability of success in one trial (say) p, then probability of failure, $q = (1 - p)$.
The probability of r success in n trials = ${}^nC_r p^r q^{n-r}$
On the basis of above information, answer the following questions :

14. The probability of a man hitting a target in one fire is 1/4. Atleast n times he must fire at the target that the chances of hitting the target at least once will exceed 2/3, then the value of n is

(a) 2 (b) 4 (c) 6 (d) 8

15. In a lot containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective, is

(a) 10^{-15} (b) $\left(\frac{1}{5}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

Matrix Match Type

16. A bag contains 14 balls which are either white or black (all number of white and black balls are equally likely). Five balls are drawn at random from the bag without replacement. Match the following columns:

Column-I		Column-II	
P.	Probability that all the five balls are black is equal to	1.	3/13
Q.	If the bag contains 11 black and 3 white balls, then the probability that all five balls are black is equal to	2.	6/65
R.	If all the five balls are black, then the probability that the bag contains 11 black and 3 white balls is equal to	3.	1/6

	P	Q	R
(a)	2	1	3
(b)	1	3	2
(c)	3	1	2
(d)	2	3	1

Numerical Value Type

17. The probability that certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. If the probability that a new component will last for one year is λ, then value of 1000 λ must be

18. The sum of the mean and variance of a binomial distribution is 15 and the sum of their squares is 117. The mean of the distribution is

19. A biased coin with probability p, $0 < p < 1$ of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 1/6, then p =

20. The probability that an automobile will be stolen and found within one week is 0.0006. The probability that an automobile will be stolen is 0.0015. The probability that a stolen automobile will be found in one week is

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted .....
No. of questions correct .....
Marks scored in percentage .....

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

Exam on 17th March 2020

CLASS-XII

CBSE warm-up!

Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Practice Paper 2020

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(i) All the questions are compulsory.
(ii) This question paper contains 36 questions.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculators is not permitted.

SECTION - A

Q.1-Q.10 are multiple choice type questions. Select the correct option.

1. Evaluate : $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

(a) $3x^2 + 4$ (b) $x(5x + 8)$
(c) $3x + 4x^2$ (d) $x(3x + 4)$

2. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find the matrix $A^2 - B$.

(a) $\begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 8 \\ 13 & -6 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -8 \\ -13 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 8 \\ 13 & 6 \end{bmatrix}$

3. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

(a) $-4\hat{j} - \hat{k}$ (b) $-\hat{i} - 4\hat{j} - \hat{k}$
(c) $4\hat{j} + \hat{k}$ (d) $\hat{i} - 4\hat{j}$

4. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

(a) 1/4 (b) 3/5 (c) 2/3 (d) 2/5

5. Optimization of the objective function is a process of

(a) Maximizing the objective function
(b) Maximizing or minimizing the objective function
(c) Minimizing the objective function
(d) Finding the profit

6. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{-1}{2}\right)$.

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{8}$

7. A bag contains 3 white and 6 black balls while another bag contains 6 white and 3 black balls. A bag is selected at random and a ball is drawn. Find the probability that the ball drawn is of white colour.

(a) 3/4 (b) 5/4 (c) 1/4 (d) 1/2

8. Evaluate : $\int \frac{\sec^2 x}{2+\tan x}dx$

(a) $\log|2+\sec x|+C$ (b) $\log|2+\tan x|+C$

(c) $2+\tan x+C$ (d) $2+\sec x+C$

9. Find the equation of plane passing through the point (1, 2, 3) and the direction cosines of the normal as l, m, n.

(a) $lx+my+nz=l+2m+3n$

(b) $lx+my+nz+(l+2m+3n)=0$

(c) $lx+my+nz=\frac{1}{2}(l+2m+3n)$

(d) None of these

10. The vector equation of the plane passing through a point having position vector $2\hat{i}+3\hat{j}+4\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-2\hat{k}$ is

(a) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=-1$

(b) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=8$

(c) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=9$

(d) $\vec{r}\cdot(2\hat{i}+\hat{j}-2\hat{k})=15$

(Q.11-Q.15) Fill in the blanks.

11. Let $f=\{(1,3),(2,1),(3,2)\}$ and $g=\{(1,2),(2,3),(3,1)\}$, then $(gof)(1)$ is _____.

12. Differentiation of $\sqrt{\cot^{-1}\sqrt{x}}$ w.r.t. x is _____.

OR

The number of discontinuous functions $y(x)$ on $[-2, 2]$ satisfying $x^2+y^2=4$ is _____.

13. If $\begin{pmatrix}2x+y & 3y\\ 0 & 4\end{pmatrix}=\begin{pmatrix}6 & 0\\ 6 & 4\end{pmatrix}'$, then $x=$ _____.

14. The interval in which the function $y=x^3+5x^2-1$ is decreasing, is _____.

OR

The slope of the normal to the curve $y^2=8x$ which passes through the point(2, 4) is _____.

15. The projection of vector $7\hat{i}+\hat{j}-4\hat{k}$ on the vector $2\hat{i}+6\hat{j}+3\hat{k}$ is _____.

(Q.16-Q.20) Answer the following questions.

16. If the equations $ax+4y+z=0$, $bx+3y+z=0$, $cx+2y+z=0$ have non-trivial solution, then find the value of $a-2b+c$.

17. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+x^4=0$.

OR

Find the order of the differential equation whose general solution is given by $y=(A+B)\cos(x+C)+De^x$.

18. Find the value of $\int_{-\pi/2}^{\pi/2}|\sin x|dx$.

19. Evaluate : $\int_0^{\pi/2} x\cos x\,dx$

20. Evaluate : $\int\cos^3 x\sin x\,dx$

SECTION - B

21. Find the range of the function

$$f(x)=\sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)....\infty}}}.$$

OR

Prove that : $3\sin^{-1}x=\sin^{-1}(3x-4x^3)$, $x\in\left[-1/2,1/2\right]$

22. Differentiate $\tan^{-1}\left(\frac{x^{1/3}+a^{1/3}}{1-x^{1/3}a^{1/3}}\right)$ w.r.t. x.

23. The total cost $C(x)$ associated with provision of free mid-day meals to x students of a school in primary classes is given by $C(x)=0.005x^3-0.02x^2+30x+50$. If the marginal cost is given by rate of change of total cost, write the marginal cost of food for 300 students.

24. If the vectors $\vec{a}=i+3j+k$, $b=2i-j-k$ and $\vec{c}=\lambda\hat{j}+3\hat{k}$ are coplanar, then find the value of λ.

25. Find the angle between the line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$.

OR

Find the direction cosines of the line passing through the points (–2, 4, –5) and (1, 2, 3).

26. A box contains N coins, of which m are fair and the rest are biased. The probability of getting head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. Find the probability that the coin drawn is fair.

SECTION - C

27. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a function defined by, $f(x)=\dfrac{x-1}{x-2}$, show that f is one-one and onto. Hence find f^{-1}.

28. For what value of a is the function f defined by

$$f(x)=\begin{cases} a\sin\dfrac{\pi}{2}(x+1), & x\le 0 \\ \dfrac{\tan x-\sin x}{x^3}, & x>0 \end{cases}$$ continuous at $x = 0$?

OR

If $x^m y^n = (x + y)^{m+n}$, prove that $\dfrac{d^2y}{dx^2}=0$.

29. Find the general solution of the differential equation

$$(x-y)\frac{dy}{dx}=x+2y.$$

30. Evaluate : $\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx$

31. Out of a group of 30 honest people, 20 always speak the truth. Two persons are selected at random from the group. Find the probability distribution of the number of selected persons who speak the truth. Also find the mean of the distribution.

OR

In a hockey match, both teams A and B scored same number of goals up to the end of the game, so as to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match.

32. A manufacturer has three machines, I, II and III installed in his factory. Machines I and II are capable of being operated for atmost 12 hours whereas machine III must be operated for atleast 5 hours a day. He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit each of M and N on the three machines are given in the following table :

Items	Number of hours required on Machine		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on items M and N respectively. How many of each item should he produce so as to maximise his profit assuming that he can sell all the items that he produced? What will be the maximum profit?

SECTION - D

33. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

34. Using integration, find the area of the triangular region whose sides have equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

OR

Find the area of the region $\{(x, y) : y^2 \le 6ax$ and $x^2 + y^2 \le 16a^2\}$ using method of integration.

35. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $4r/3$. Also find maximum volume in terms of volume of the sphere.

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank?

36. Find the cartesian as well as vector equations of the planes through the intersection of the planes $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at unit distance from origin.

SOLUTIONS

1. (b) : We have, $\begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix} = x(5x+1)+7(x)$

$= 5x^2 + x + 7x = 5x^2 + 8x = x(5x + 8)$

2. (a) :

$$A^2=\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}=\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix}=\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$$

$$\therefore \quad A^2 - B = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$$

3. (a) : The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,

$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$\therefore$ Required sum $= \vec{a} + \vec{b} + \vec{c}$

$= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k}) = -4\hat{j} - \hat{k}$

4. (d) : We have, $P(A) = 1/4$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ ($\because$ A, B are independent)

$= 1/4 + P(B) - (1/4)\,P(B) = 2P(B) - 1/4$ (Given)

$\Rightarrow \quad P(B) = 2/5$

5. (b)

6. (a) : Principal value of $\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{-1}{2}\right)$

$$= \frac{2\pi}{3} + 2\left(\frac{-\pi}{6}\right) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$$

7. (d) : Let E_1 = event that bag I is selected

E_2 = event that bag II is selected

E = event that the ball drawn is of white colour

By rule of total probability,

$P(E) = P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2)$

$$= \frac{1}{2} \cdot \frac{3}{9} + \frac{1}{2} \cdot \frac{6}{9} = \frac{9}{18} = \frac{1}{2}$$

8. (b) : Let $I = \int \frac{\sec^2 x}{2 + \tan x}\,dx$

Put $2 + \tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore \quad I = \int \frac{dt}{t} = \log|t| + C = \log|2 + \tan x| + C$$

9. (a) : Equation of plane passing through (1, 2, 3) having direction cosines of normal as l, m, n is

$l(x - 1) + m(y - 2) + n(z - 3) = 0$

$\Rightarrow \quad lx + my + nz = l + 2m + 3n$

10. (a) : Vector equation of plane passing through a point having position vector $\vec{a}$ and perpendicular to $\vec{n}$ is given by $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

Here $\vec{a} = 2i + 3j + 4k$, $\vec{n} = 2i + j - 2k$

$\therefore$ Required equation $= \vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 4 + 3 - 8 = -1$

11. Since $f = \{(1, 3), (2, 1), (3, 2)\}$

$\therefore \quad f(1) = 3, f(2) = 1$ and $f(3) = 2$ ...(i)

Again $g = \{(1, 2), (2, 3), (3, 1)\}$

$\Rightarrow \quad g(1) = 2, g(2) = 3$ and $g(3) = 1$ ...(ii)

Using (i) and (ii), we have $(gof)\,(1) = g(f\,(1)) = g(3) = 1$

12. Let $y = \sqrt{\cot^{-1}\sqrt{x}}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{\cot^{-1}\sqrt{x}}} \times \left(-\frac{1}{1+x}\right) \times \frac{1}{2\sqrt{x}}$$

$$= \frac{-1}{4(\sqrt{\cot^{-1}\sqrt{x}})(1+x)\sqrt{x}}$$

OR

Functions which satisfy the relation $x^2 + y^2 = 4$ are $y(x) = \sqrt{4 - x^2}$ and $y(x) = -\sqrt{4 - x^2}$.

And both functions are continuous in $[-2, 2]$.

13. $\because \begin{pmatrix} 2x + y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 6 \\ 0 & 4 \end{pmatrix}$

$\therefore \quad 2x + y = 6$ and $3y = 6 \Rightarrow y = 2$

Putting the value of y, we get

$2x + 2 = 6 \Rightarrow 2x = 4 \Rightarrow x = 2$

14. Given, $y = x^3 + 5x^2 - 1 \Rightarrow \frac{dy}{dx} = x(3x + 10)$

For function to be decreasing, $\frac{dy}{dx} < 0$

$$\therefore \quad x(3x + 10) < 0 \Rightarrow \frac{-10}{3} < x < 0$$

OR

We have, $y^2 = 8x$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{8}{2y} \Rightarrow \left(\frac{dy}{dx}\right)_{(2,4)} = \frac{8}{2 \times 4} = 1$$

$\therefore$ Slope of normal $= -1$

15. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{(7\hat{i} + \hat{j} - 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

16. For non-trivial solution, $\begin{vmatrix} a & 4 & 1 \\ b & 3 & 1 \\ c & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad a(3 - 2) - 4(b - c) + 1(2b - 3c) = 0 \Rightarrow a - 2b + c = 0$

17. Order = 2, Degree = 2

$\therefore$ Order + Degree = 2 + 2 = 4

OR

Given $y = (A + B)\cos(x + C) + De^x$

or $y = k\cos(x + C) + De^x$

Now order of a differential equation is same as the number of arbitrary unknowns present in the solution.

Hence order of differential equation is 3.

18. $\because$ $|\sin x|$ is an even function.

$$\therefore \quad \int_{-\pi/2}^{\pi/2} |\sin x|\,dx = 2\int_{0}^{\pi/2} |\sin x|\,dx = 2\int_{0}^{\pi/2} \sin x\,dx = 2$$

19. $\int_0^{\pi/2} x\cos x\,dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\sin x\,dx$

$= \frac{\pi}{2} + [\cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$

20. Let $I = \int \cos^3 x \sin x\,dx$

Put $\cos x = t \Rightarrow \sin x\,dx = -dt$

$\therefore \quad I = -\int t^3 dt = -\frac{t^4}{4} + C = -\frac{1}{4}\cos^4 x + C$

21. Given, $f(x) = \sqrt{(1-\cos x)\sqrt{(1-\cos x)\sqrt{(1-\cos x)...\infty}}}$

$= (1-\cos x)^{1/2}\,(1-\cos x)^{1/4}\,(1-\cos x)^{1/8}\,.....\infty$

$= (1-\cos x)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\infty} = (1-\cos x)^{\frac{1/2}{1-1/2}} = 1-\cos x$

$\Rightarrow$ Range of $f(x)$ is $[0, 2]$.

OR

Put $\sin^{-1}x = \theta$. Then $x = \sin\theta$

Now, $\sin 3\theta = (3\sin\theta - 4\sin^3\theta) = (3x - 4x^3)$

$\Rightarrow \quad 3\theta = \sin^{-1}(3x - 4x^3)$

$\Rightarrow \quad 3\sin^{-1}x = \sin^{-1}(3x - 4x^3) \quad [\because\ \theta = \sin^{-1}x]$

Hence, $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

22. Let $y = \tan^{-1}\left(\frac{x^{1/3} + a^{1/3}}{1 - x^{1/3}\,a^{1/3}}\right) = \tan^{-1}x^{1/3} + \tan^{-1}a^{1/3}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{1+(x^{1/3})^2}\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) = \frac{1}{3x^{2/3}\,(1+x^{2/3})}$

23. Given, $C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$

Marginal cost $= \frac{dC}{dx} = 0.015x^2 - 0.04\,x + 30$

Now, $\left(\frac{dC}{dx}\right)_{x=300} = 0.015 \times 300^2 - 0.04 \times 300 + 30$

$= 1350 - 12 + 30 = 1368$

24. Since the vectors are coplanar.

$\therefore \quad \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0$

$\Rightarrow \quad 1(-3+\lambda) - 3\,(6-0) + 1(2\lambda - 0) = 0$

$\Rightarrow \quad -3 + \lambda - 18 + 2\lambda = 0 \Rightarrow 3\lambda - 21 = 0 \Rightarrow \lambda = 7$

25. Let θ be the angle between the given line and the plane.

$\therefore \quad \sin\theta = \frac{2+1+1}{\sqrt{3}\,\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

OR

Let coordinates of P are $(-2, 4, -5)$ and Q are $(1, 2, 3)$.

So, $PQ = \sqrt{(1-(-2))^2 + (2-4)^2 + (3-(-5))^2} = \sqrt{77}$

Thus, the direction cosines of the line joining two points are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$.

26. Let E be the event that the coin tossed twice shows first head and then tail and F be the event that the coin drawn is fair.

$P(F/E) = \frac{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{N-m}{N}\cdot\frac{2}{3}\cdot\frac{1}{3}} = \frac{9m}{m+8N}$

27. Here, $f: A \to B$ is given by $f(x) = \frac{x-1}{x-2}$,

where $A = R - \{2\}$ and $B = R - \{1\}$

Let $f(x_1) = f(x_2)$, where $x_1, x_2 \in A$ (*i.e.*, $x_1 \neq 2, x_2 \neq 2$)

$\Rightarrow \quad \frac{x_1-1}{x_1-2} = \frac{x_2-1}{x_2-2} \Rightarrow (x_1-1)(x_2-2) = (x_1-2)(x_2-1)$

$\Rightarrow \quad x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - x_1 - 2x_2 + 2$

$\Rightarrow \quad x_1 = x_2 \Rightarrow f$ is one-one.

Let $y \in B = R - \{1\}$ *i.e.*, $y \in R$ and $y \neq 1$ such that $f(x) = y$

$\Leftrightarrow \quad \frac{x-1}{x-2} = y \Leftrightarrow (x-2)y = x-1 \Leftrightarrow x = \frac{2y-1}{y-1} \qquad ...(i)$

$\therefore \quad f(x) = y$ when $x = \frac{2y-1}{y-1} \in A$ (as $y \neq 1$)

Hence, f is onto.

Thus, f is one–one and onto.

From (i), $f^{-1}: B \to A$ is given by $x = f^{-1}(y)$

i.e., $f^{-1}(y) = \frac{2y-1}{y-1}$.

28. $\because$ $f(x)$ is continuous at $x = 0$,

$\therefore \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Here, $f(0) = a\sin\frac{\pi}{2} = a$

$\therefore \quad \lim_{x\to 0^-} f(x) = \lim_{h\to 0} a\,\sin\left(\frac{\pi}{2}(-h+1)\right)$

$= \lim_{h\to 0} a\sin\left(\frac{\pi}{2} - h\frac{\pi}{2}\right) = \lim_{h\to 0} a\cos\left(\frac{\pi}{2}h\right) = a$

$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{\tan h - \sin h}{h^3} = \lim_{h\to 0} \frac{\sin h\left(\frac{1}{\cos h} - 1\right)}{h^3}$

$= 1 \times \lim_{h\to 0}\frac{1}{\cos h} \times \lim_{x\to 0}\frac{2\sin^2\left(\frac{h}{2}\right)}{4\times\left(\frac{h}{2}\right)^2} = 1\times\frac{1}{2} = \frac{1}{2} \Rightarrow \quad a = \frac{1}{2}$

Hence, $f(x)$ is continuous at $x = 0$, if $a = 1/2$.

OR

Refer to answer 82, Page no. 113 of MTG CBSE Champion Mathematics, Class-12.

29. We have, $(x-y)\frac{dy}{dx}=x+2y \Rightarrow \frac{dy}{dx}=\frac{x+2y}{x-y}$ ...(i)

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$$v+x\frac{dv}{dx}=\frac{x+2vx}{x-vx}=\frac{1+2v}{1-v} \Rightarrow x\frac{dv}{dx}=\frac{1+v+v^2}{1-v}$$

$$\Rightarrow \frac{1-v}{1+v+v^2}dv=\frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{-\frac{1}{2}(2v+1)+\frac{3}{2}}{v^2+v+1}dv=\log x+C$$

$$\Rightarrow -\frac{1}{2}\int\frac{2v+1}{v^2+v+1}dv+\frac{3}{2}\int\frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\log x+C$$

$$\Rightarrow -\frac{1}{2}\log\left(v^2+v+1\right)+\frac{3}{2}\cdot\frac{1}{\sqrt{3}/2}\tan^{-1}\left[\frac{v+\frac{1}{2}}{\sqrt{3}/2}\right]=\log x+C$$

$$\Rightarrow -\frac{1}{2}\log\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right)+\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}\cdot x}\right)=\log x+C$$

$$\Rightarrow -\frac{1}{2}\log(y^2+xy+x^2)+\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}\cdot x}\right)=C$$

30. Let $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx=\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx$ ...(i)

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}dx=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\sin x}dx \quad ...(ii)$$

Adding (i) and (ii), we get

$$2I=\int_0^{\pi}\frac{\pi\sin x}{1+\sin x}dx \Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x-\sin^2 x}{\cos^2 x}dx=\frac{\pi}{2}\int_0^{\pi}\left(\frac{\sin x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\right)dx$$

$$=\frac{\pi}{2}\int_0^{\pi}(\tan x\sec x-\tan^2 x)\,dx=\frac{\pi}{2}[\sec x-\tan x+x]_0^{\pi}$$

$$=-\frac{\pi}{2}+\frac{\pi^2}{2}-\frac{\pi}{2}=\frac{\pi^2}{2}-\pi=\frac{\pi}{2}(\pi-2)$$

31. Total number of honest people = 30

The number of people who speak truth = 20

The number of people who do not speak truth = 10

Number of selected persons = 2

Let X denotes the number of people who speak truth.

$\therefore$ X can take values 0, 1, 2

Now, $P(X=0)=\frac{{}^{20}C_0\times{}^{10}C_2}{{}^{30}C_2}=\frac{10\times9}{30\times29}=\frac{9}{87}$

$$P(X=1)=\frac{{}^{20}C_1\times{}^{10}C_1}{{}^{30}C_2}=\frac{20\times10\times2}{30\times29}=\frac{40}{87}$$

$$P(X=2)=\frac{{}^{20}C_2\times{}^{10}C_0}{{}^{30}C_2}=\frac{20\times19}{30\times29}=\frac{38}{87}$$

Hence, the probability distribution of X is

X	0	1	2
$P(X)$	$\frac{9}{87}$	$\frac{40}{87}$	$\frac{38}{87}$

Mean of the distribution

$$\bar{X}=0\times\frac{9}{87}+1\times\frac{40}{87}+2\times\frac{38}{87}=\frac{40+76}{87}=\frac{116}{87}=\frac{4}{3}$$

OR

Refer to answer 20, Page no. 331 of MTG CBSE Champion Mathematics, Class-12.

32. Let x and y be the number of items of M and N produced by the three machines respectively.

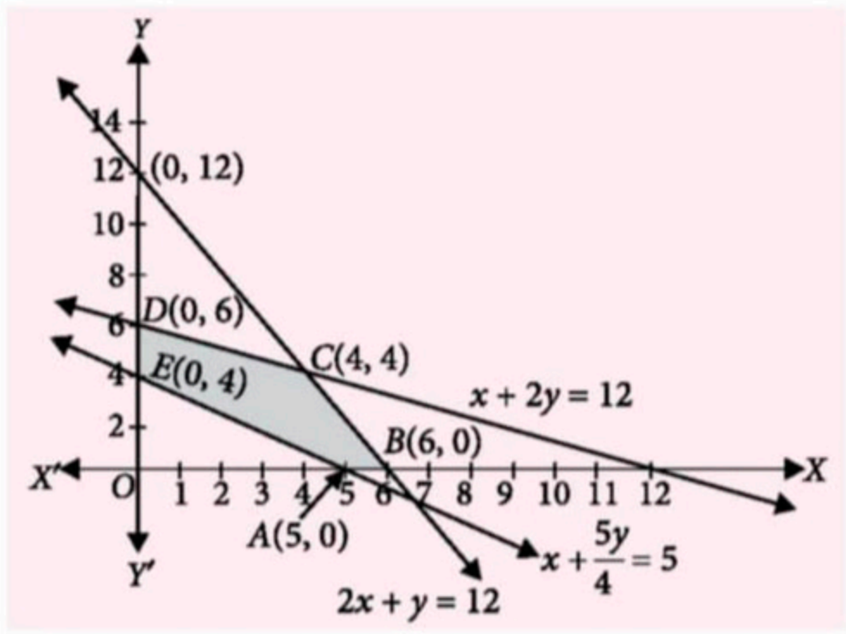
Mathematical formulation of the given problem is as follows :

Maximise, $Z = 600x + 400y$

subject to the constraints :

$$x+2y\le12;\ 2x+y\le12;\ x+\frac{5}{4}y\ge5;\ x,y\ge0$$

Let us draw the graph of constraints. $ABCDEA$ is the feasible region (shaded) as shown in the figure. Observe that the feasible region is bounded, and coordinates of the corner points A, B, C, D and E are (5, 0), (6, 0), (4, 4), (0, 6) and (0, 4) respectively.



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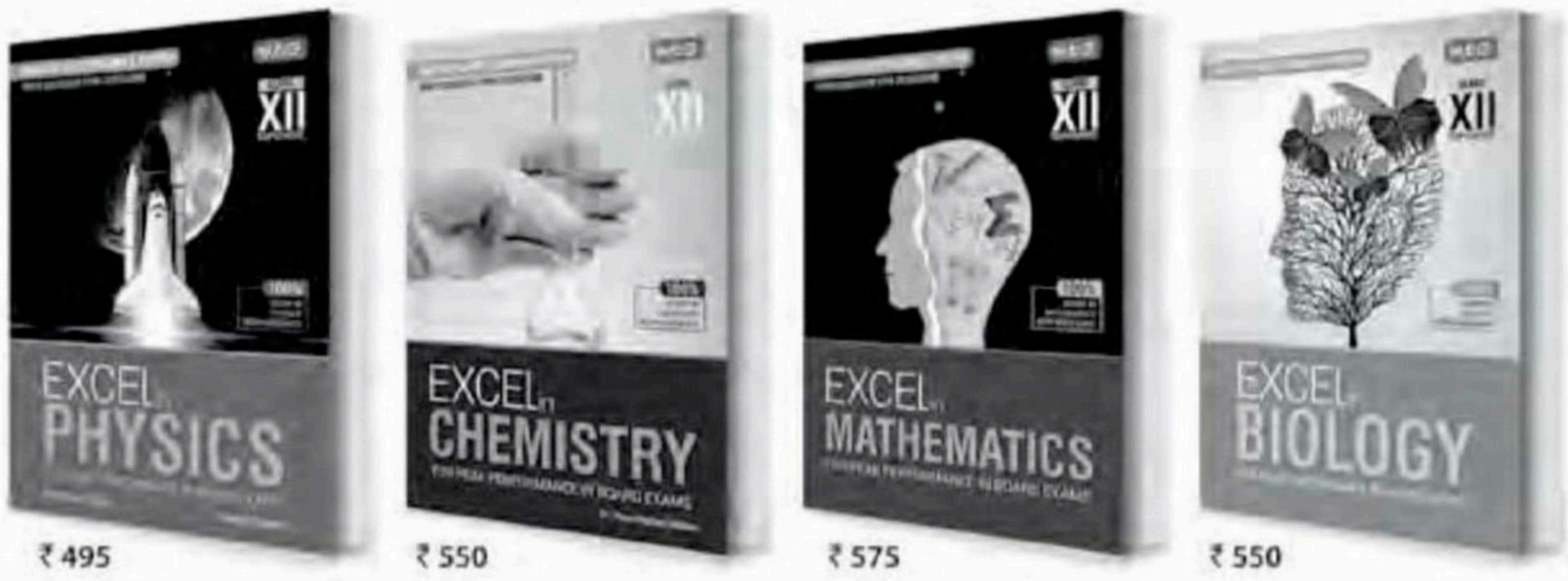

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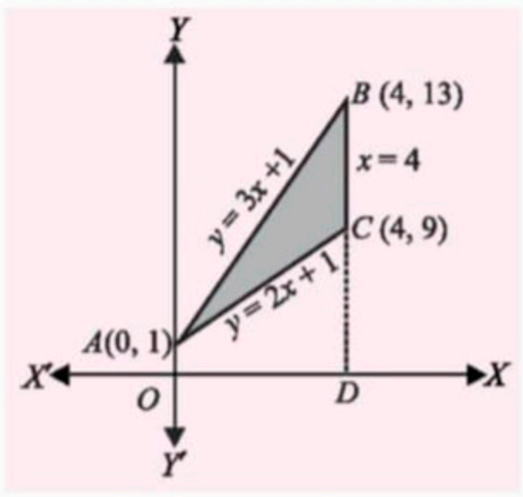
Let us evaluate $Z = 600x + 400y$ at these corner points.

Corner Points	$Z = 600x + 400y$
$A(5, 0)$	3000
$B(6, 0)$	3600
$C(4, 4)$	4000 (Maximum)
$D(0, 6)$	2400
$E(0, 4)$	1600

We clearly see that the point (4, 4) is giving the maximum value of Z. Hence, the manufacturer has to produce 4 units of each item to get the maximum profit of ₹ 4000.

33. *Refer to answer 65, Page no. 71 of MTG CBSE Champion Mathematics, Class-12.*

34. The given lines are
$y = 2x + 1$ ...(i)
$y = 3x + 1$ ...(ii) and
$x = 4$ ...(iii)
Solving (i) and (ii), we get $x = 0, y = 1$
Solving (ii) and (iii), we get $x = 4, y = 13$
Solving (iii) and (i), we get $x = 4, y = 9$



$\therefore$ Required area = area (ΔABC)
= area ($OABD$) – area ($OACD$)

$$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx = \left[\frac{3x^2}{2} + x\right]_0^4 - \left[x^2 + x\right]_0^4$$

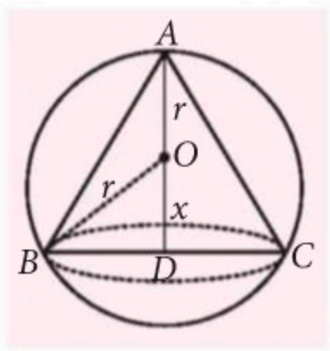
$$= \left[\left(\frac{3}{2}(16) + 4\right) - (0+0)\right] - [(16+4) - (0+0)]$$

= 28 – 20 = 8 sq. units

OR

Refer to answer 35, Page no. 211 of MTG CBSE Champion Mathematics, Class-12.

35. Let ABC be a cone of maximum volume inscribed in the sphere.
Let $OD = x$ and $OB = OA = r$



$\therefore \quad BD = \sqrt{r^2 - x^2}$
and $AD = AO + OD$
$= r + x$ = altitude of cone.
Let V be the volume of cone.

$$V = \frac{1}{3}\pi (BD)^2 (AD) = \frac{1}{3}\pi (r^2 - x^2)(r + x)$$

$$\therefore \quad \frac{dV}{dx} = \frac{1}{3}\pi\left[(r^2 - x^2) + (r + x)(-2x)\right]$$

$$= \frac{\pi}{3}\left[r^2 - 3x^2 - 2rx\right] \text{ and } \frac{d^2V}{dx^2} = \frac{\pi}{3}\left[-6x - 2r\right]$$

For maximum or minimum value $\frac{dV}{dx} = 0$

$$\Rightarrow \quad r^2 - 3x^2 - 2rx = 0 \Rightarrow r^2 - 3rx + rx - 3x^2 = 0$$

$$\Rightarrow \quad (r - 3x)(r + x) = 0 \Rightarrow x = \frac{r}{3} \qquad [\because \ r + x \neq 0]$$

$$\text{Also, } \left(\frac{d^2V}{dx^2}\right)_{x=\frac{r}{3}} = \frac{\pi}{3}\left[-6\left(\frac{r}{3}\right) - 2r\right]$$

$$= \frac{\pi}{3}[-2r - 2r] = \frac{-4}{3}r\pi < 0$$

$\Rightarrow$ V is maximum when $x = \frac{r}{3}$

and altitude of cone = $AD = r + x = r + \frac{r}{3} = \frac{4r}{3}$

Also, maximum volume of cone when $x = \frac{r}{3}$

$$= \frac{1}{3}\pi\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{\pi}{3}\left(\frac{8}{9}r^2\right)\left(\frac{4}{3}r\right)$$

$$= \frac{8}{27}\left(\frac{4}{3}\pi r^3\right) = \frac{8}{27} \text{ (Volume of sphere)}$$

OR

Refer to answer 69, Page no. 139 of MTG CBSE Champion Mathematics, Class-12.

36. The equation of the planes through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and
$\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ is
$[\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12] + \lambda[\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k})] = 0$

$$\Rightarrow \quad \vec{r} \cdot [(2 + 3\lambda)\hat{i} + (6 - \lambda)\hat{j} + 4\lambda\,\hat{k}] + 12 = 0 \qquad \text{...(i)}$$

These planes are at a unit distance from the origin. Therefore, length of the perpendicular from the origin on (i) is 1 unit.

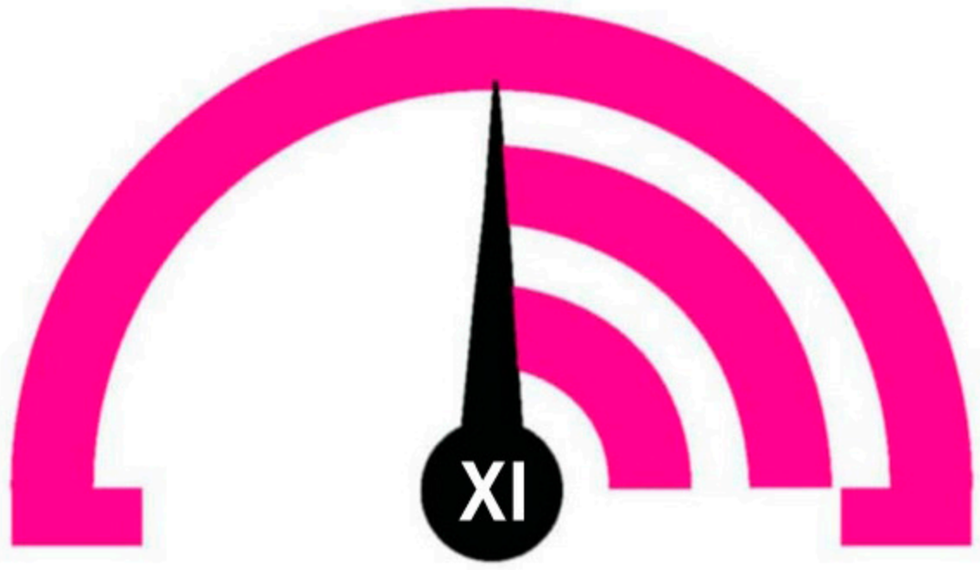
$$\Rightarrow \quad \frac{12}{\sqrt{(2 + 3\lambda)^2 + (6 - \lambda)^2 + 16\lambda^2}} = 1$$

$$\Rightarrow \quad 144 = (2 + 3\lambda)^2 + (6 - \lambda)^2 + 16\lambda^2$$

$$\Rightarrow \quad 144 = 40 + 26\lambda^2 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = \pm 2$$

Putting the values of λ in (i), we get
$\vec{r} \cdot (8\hat{i} + 4\hat{j} + 8\hat{k}) + 12 = 0$ and
$\vec{r} \cdot (-4\hat{i} + 8\hat{j} - 8\hat{k}) + 12 = 0$
as the equations of the required planes. These equations can also be written as
$\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$ and $\vec{r} \cdot (-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 10 : Statistics and Probability

Time Taken : 60 Min.

Only One Option Correct Type

1. If two variates X and Y are connected by the relation $Y = \frac{aX+b}{c}$, where a, b, c are constants such that $ac < 0$, then

(a) $\sigma_Y = \frac{a}{c}\sigma_X$ (b) $\sigma_Y = -\frac{a}{c}\sigma_X$

(c) $\sigma_Y = -\frac{a}{c}\sigma_X + b$ (d) None of these

2. In a class of 15 students, 5 of them are boys and 10 students are girls. A team of 11 members has to be formed at random. The probability that the team has at least 4 boys, is

(a) $\frac{57}{91}$ (b) $\frac{54}{95}$ (c) $\frac{54}{91}$ (d) $\frac{51}{91}$

3. The mean of the numbers a, b, 8, 5, 10, is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?

(a) $a = 1, b = 6$ (b) $a = 3, b = 4$

(c) $a = 0, b = 7$ (d) $a = 5, b = 2$

4. Of the 25 questions in a unit, a student has worked out only 20. In a sessional test of that unit, two questions were asked by the teacher. The probability that the student can solve both the questions correctly, is

(a) 8/25 (b) 17/25 (c) 9/10 (d) 19/30

5. For two data sets each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4 respectively. The variance of the combined data set is

(a) 6 (b) 13/2 (c) 5/2 (d) 11/2

6. Three randomly chosen non-negative integers x, y and z are found to satisfy the equation $x + y + z = 10$. Then the probability that z is even, is

(a) $\frac{5}{11}$ (b) $\frac{1}{2}$ (c) $\frac{36}{55}$ (d) $\frac{6}{11}$

One or More Than One Option(s) Correct Type

7. The chance (p_1) of an event happening is the square of the chance (p_2) of a second event but the odds against the first are the cube of the odds against the seconds. The chances of the events are

(a) $p_1 = \frac{1}{9}$ (b) $p_1 = \frac{1}{16}$ (c) $p_2 = \frac{1}{3}$ (d) $p_2 = \frac{1}{4}$

8. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square, then

(a) $p_1 = p_2$ (b) $p_1 < p_2$

(c) $p_1 > p_2$ (d) $p_1^2 - p_2^2 < \frac{1}{3}$

9. Let X be a set containing n elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is

(a) $\frac{{}^{2n}C_n}{2^{2n}}$ (b) $\frac{1}{{}^{2n}C_n}$

(c) $\frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n \cdot n!}$ (d) $\frac{3^n}{4^n}$

10. Three six faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $k (3 \le k \le 8)$ is

(a) $\frac{(k-1)(k-2)}{432}$ (b) $\frac{k(k-2)}{432}$

(c) ${}^{k-1}C_2 \times \frac{1}{216}$ (d) $\frac{k^2}{432}$

11. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively

(a) 32, 4 (b) 28, 2 (c) 28, 4 (d) 32, 2

12. If A and B are two events, the probability that exactly one of them occurs is given by

(a) $P(A) + P(B) - 2P(A \cap B)$
(b) $P(A \cap B') + P(A' \cap B)$
(c) $P(A \cup B) - P(A \cap B)$
(d) $P(A') + P(B') - 2P(A' \cap B')$

13. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then

(a) $P(A \cup B) \geq \frac{3}{4}$ (b) $P(A' \cap B) \leq \frac{1}{4}$

(c) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$ (d) None of these

Comprehension Type

If n positive integers taken at random and multiplied together, then the chance that the last digit of the product would be

14. 1, 3, 5, 7 or 9 is

(a) $\left(\frac{2}{5}\right)^n$ (b) $\left(\frac{1}{2}\right)^n$ (c) $\frac{2^n - 1}{5^n}$ (d) $\frac{5^n - 4^n}{10^n}$

15. 5 is

(a) $\frac{5^n - 4^n}{10^n}$ (b) $\frac{2^n - 1}{5^n}$

(c) $\frac{4^n - 2^n}{5^n}$ (d) $\left(\frac{2}{5}\right)^n$

Matrix Match Type

16. Match the following columns:

	Column-I	Column-II	
P.	If the coefficient of skewness of a distribution is 0.32. If its standard deviation is 6.5 and mean is 29.6, then the mode of the distribution is given by	1.	10
Q.	In a series of $2n$ observations, half of them equal to a and remaining half equal to $-a$. If the standard deviation of the observations is 2, then $\lvert a \rvert$ =	2.	2
R.	If the mean of 10 observations is 50 and the sum of the squares of the deviations of the observations from the mean is 250, then the coefficient of variation of those observations is	3.	27.5

	P	Q	R
(a)	2	1	3
(b)	1	3	2
(c)	3	2	1
(d)	3	1	2

Numerical Value Type

17. A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is ₹ 54 and per day mean wage of all the workers is ₹ 60, then per day mean wage of the night shift workers (in ₹) is

18. The probabilities of two events A and B are 0.3 and 0.6 respectively. The probability that both A and B occur simultaneously is 0.18. Then, the probability that neither A nor B occurs is

19. Suppose a population A has 100 observations 101, 102, ..., 200, and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two populations, respectively, then V_A / V_B is

20. Let A, B, C be three events. If the probability of occurring exactly one event out of A and B is $1 - a$, out of B and C is $1 - 2a$, out of C and A is $1 - a$ and that of occurring three events simultaneously is a^2. If the probability that atleast one out of A, B, C will occur is greater than λ, then the value of 3λ must be

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No. of questions correct .....
Marks scored in percentage .....

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90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

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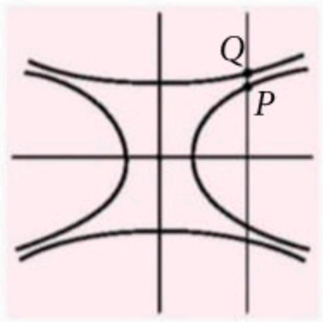
1. A straight line is drawn parallel to conjugate axis of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to meet it and the conjugate hyperbola in the points P and Q. Show that the normals at P and Q meet on the x-axis. Also find the locus of point of intersection of tangents at P and Q.

(Ahan Bandyopadhyay, Gwalior)

Ans. Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore$ The equation of conjugate hyperbola is $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Since point P and Q are symmetric about x-axis.

$\therefore$ Co-ordinates of P are $(a\tan\theta, b\sec\theta)$

and co-ordinates of Q are $(a\tan\theta, -b\sec\theta)$

For hyperbola:

Slope of tangent at $P=\frac{b\tan\theta}{a\sec\theta}$

$\therefore$ Equation of tangent at P is $\frac{y\sec\theta}{b}-\frac{x\tan\theta}{a}=1$ ...(i)

and equation of normal at P is

$\frac{by}{\sec\theta}+\frac{ax}{\tan\theta}=a^2+b^2$ ...(ii)

For conjugate hyperbola:

Slope of tangent at $Q=-\frac{b\tan\theta}{a\sec\theta}$

$\therefore$ Equation of tangent at Q is $-\frac{y\sec\theta}{b}-\frac{x\tan\theta}{a}=1$ ...(iii)

and equation of normal at Q is $-\frac{by}{\sec\theta}+\frac{ax}{\tan\theta}=a^2+b^2$ ...(iv)

Subtracting (iv) from (ii), we get

$\frac{2by}{\sec\theta}=0\Rightarrow y=0$

$\therefore$ Normals at P and Q meet on the x-axis.

Subtracting (iii) from (i), we get $\frac{2y\sec\theta}{b}=0\Rightarrow y=0$

From (iii), $x=-\frac{a}{\tan\theta}$

Hence, required locus is $y=0$.

2. Find the general solution of $2^{\sin x}+2^{\cos x}=2^{1-\frac{1}{\sqrt{2}}}$.

(Richa Sharma, Punjab)

Ans. Using A.M. - G.M. inequality, we have

$\frac{2^{\sin x}+2^{\cos x}}{2}\geq\sqrt{2^{\sin x}\cdot 2^{\cos x}}$

$\Rightarrow\quad 2^{\sin x}+2^{\cos x}\geq 2\cdot\sqrt{2^{\sin x+\cos x}}$

But minimum value of $\cos x+\sin x$ is $-\sqrt{2}$.

$\therefore\quad 2^{\sin x}+2^{\cos x}\geq 2\cdot\sqrt{2^{-\sqrt{2}}}=2^{1-\frac{1}{\sqrt{2}}}$

But the given equation is $2^{\sin x}+2^{\cos x}=2^{1-\frac{1}{\sqrt{2}}}$, which can hold only if $2^{\sin x}=2^{\cos x}$.

$\Rightarrow\quad \sin x=\cos x\Rightarrow\tan x=1\Rightarrow x=n\pi+\pi/4$

3. Solve for real x : $4^x-3^{(x-1/2)}=3^{(x+1/2)}-2^{(2x-1)}$

(Rajat Shukla, Delhi)

Ans. We have, $4^x-\frac{3^x}{\sqrt{3}}=3^x\cdot\sqrt{3}-\frac{4^x}{2}$

$\Rightarrow\quad 4^x\left(1+\frac{1}{2}\right)=3^x\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$\Rightarrow\quad 4^x\left(\frac{3}{2}\right)=3^x\left(\frac{3+1}{\sqrt{3}}\right)\Rightarrow\frac{4^x}{3^x}=\frac{8}{3\sqrt{3}}$

$\Rightarrow\quad \left(\frac{4}{3}\right)^x=\left(\frac{4}{3}\right)^{3/2}\Rightarrow x-\frac{3}{2}=0\Rightarrow x=\frac{3}{2}$

EXAM CORNER 2020

Exam	Date
JEE Main II	05th, 07th to 09th & 11th April
SRMJEEE	12th to 20th April
VITEEE	13th to 19th April
J&K CET	19th April
Kerala PET	20th & 21st April
Karnataka CET	22nd & 23rd April
COMEDK (Engg.)	10th May
AMU (Engg.)	10th May
UPSEE	10th May
MHT CET	16th to 20th May
BITSAT	16th to 25th May
JEE Advanced	17th May

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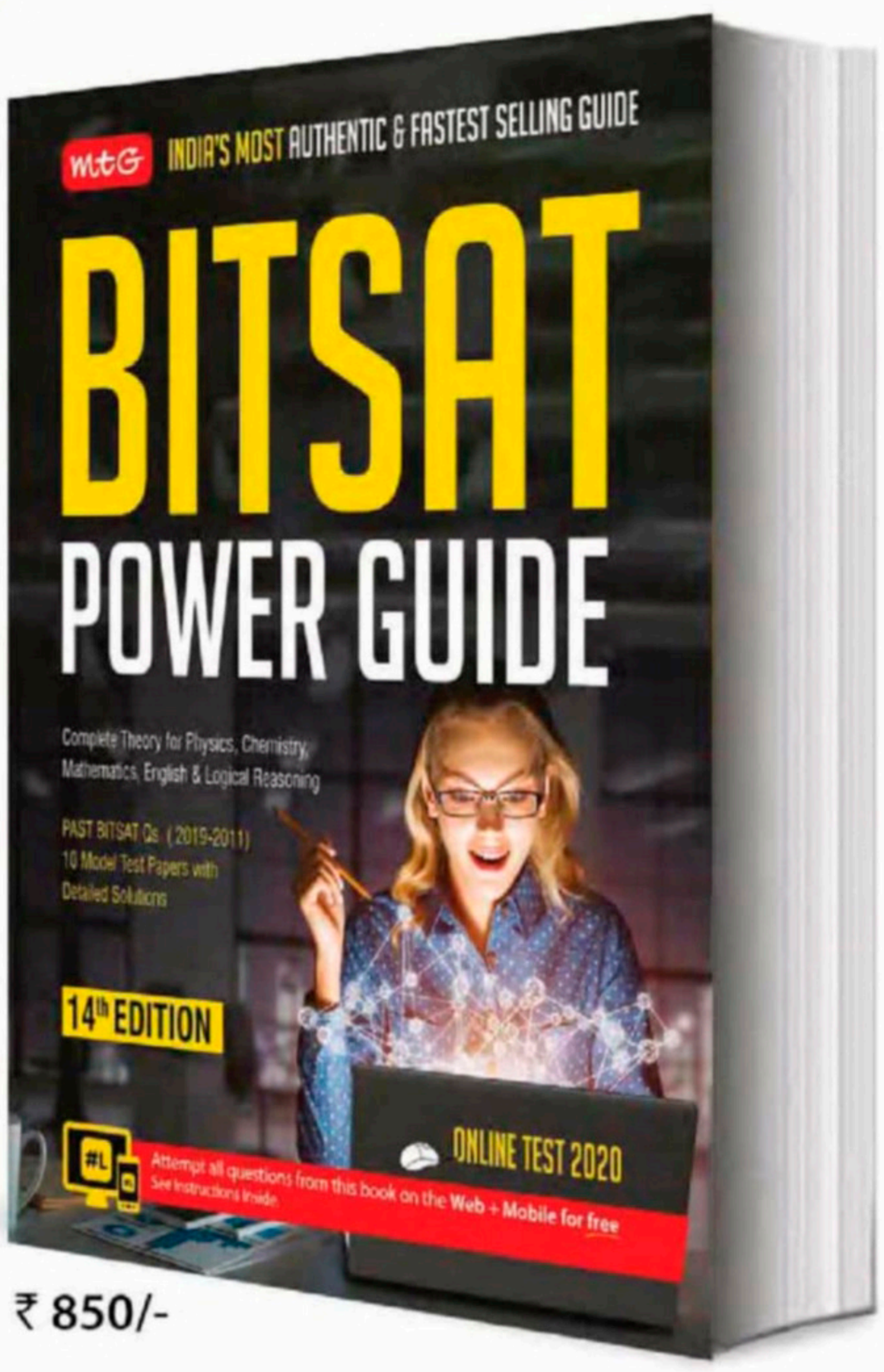

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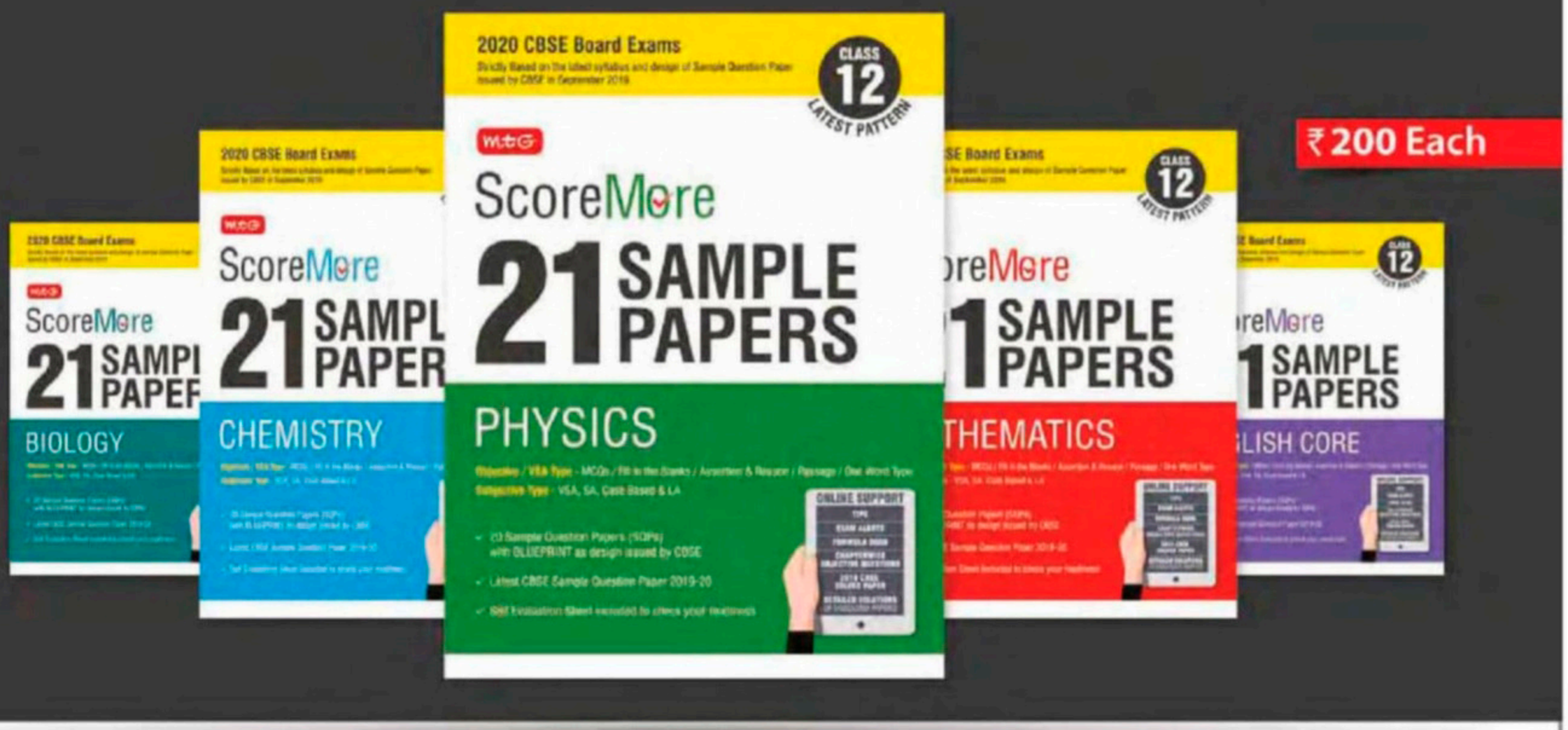

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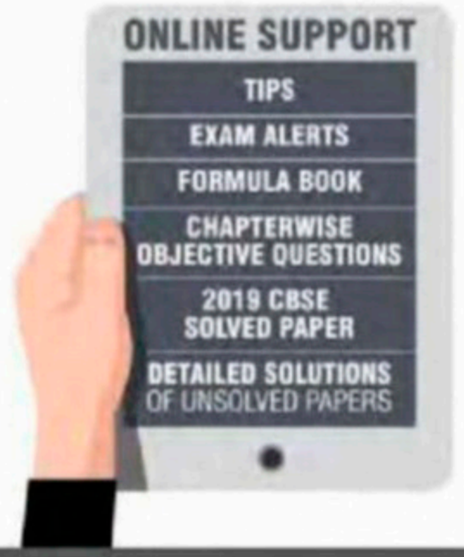

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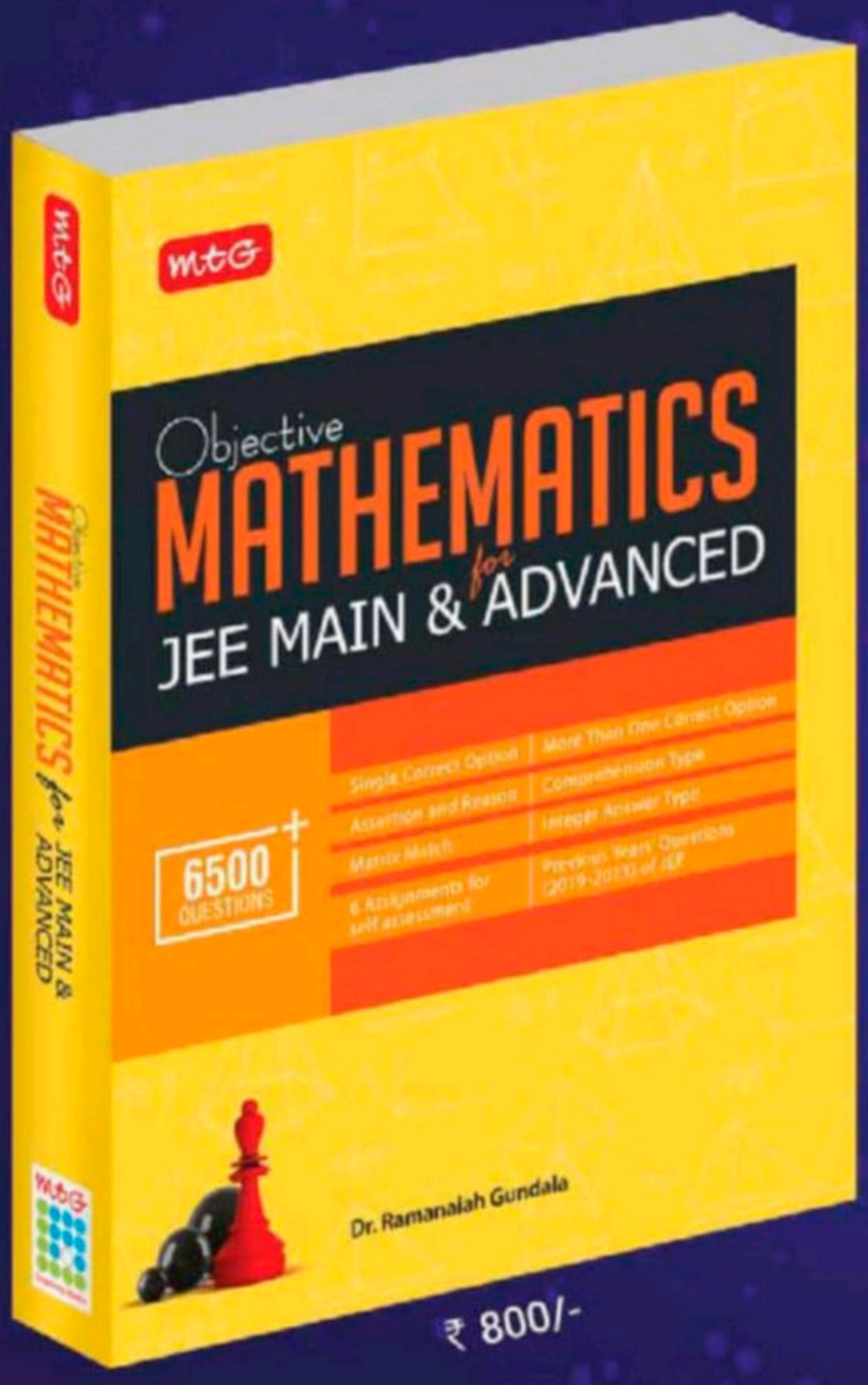

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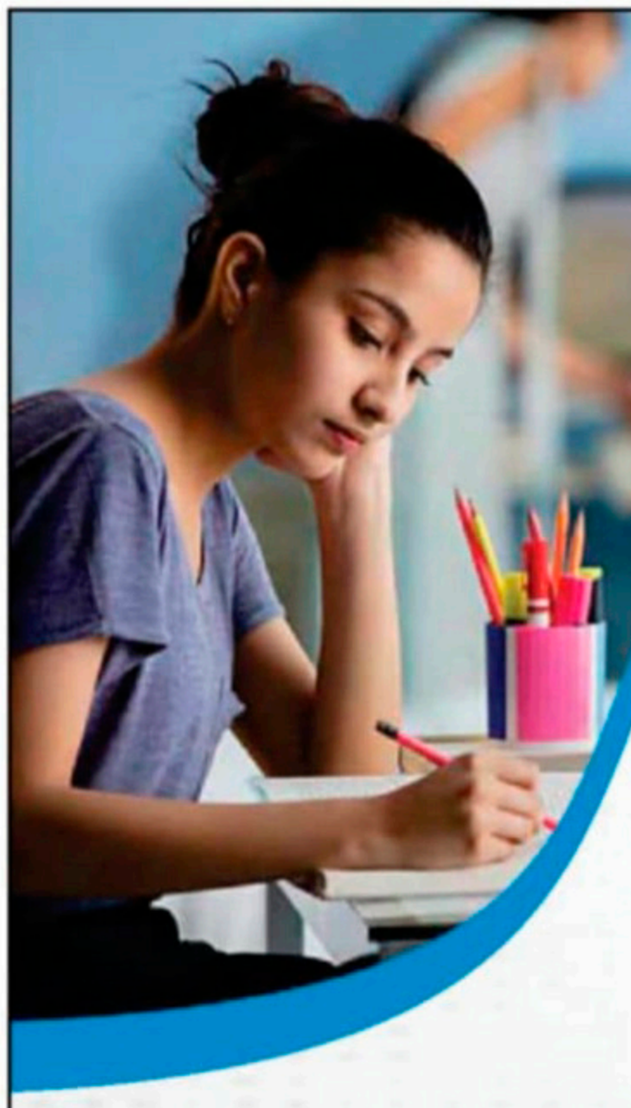

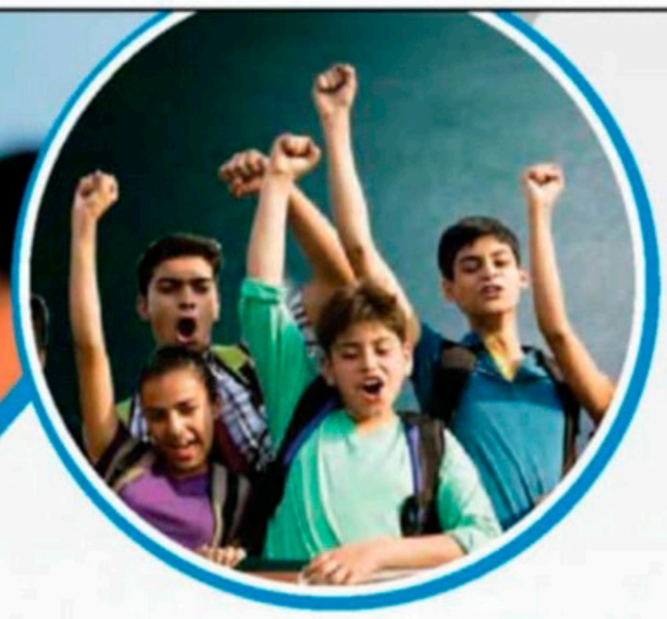

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